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A

KEY

TO

CHASE'S
COMMON SCHOOL ARITHMETIC,

WITH

EXPLANATIONS AND REMARKS UPON THE PECULIAR
FEATURES OF THE WORK,

AND

OPERATIONS OF THE MORE DIFFICULT EXAMPLES.

BY A. P. STONE.

WORCESTER:
PUBLISHED BY A. HUTCHINSON.
1858.

L. Auc. T. 118.53.357

3 2044 096 994 868

A

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Educ T 113.52.527

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PREFACE.

THE following Key has been prepared for the purpose of furnishing a fuller explanation of some of the more difficult portions of Chase's Common School Arithmetic than could be given in the body of the book without rendering it objectionable on account of size and prolixity, and also for the purpose of calling attention to some of the peculiar features and excellences of that work. So long as the present method of teaching mathematics prevails, and text books are prepared upon the principles which, at present, govern their construction, there will often be found a necessity for a more extended elucidation of principles, rules, and examples than is convenient or desirable to incorporate into a text book for the use of pupils. Especially will this be true if the work is an elementary one, where great prominence should be given to leading principles and their applications, unencumbered with diffuse explanations and unnecessary details, which rather serve to prevent the learner from obtaining a clear and ready perception of the thing to be acquired. In the study of arithmetic, it is also oftentimes desirable, on the part of both teacher and pupil, to have the answers to the examples in a separate work, whether they are or are not inserted in the text book. To meet these wants, to some extent at least, has been the object of the following pages.

An experience of nearly four years in the use of the Common School Arithmetic, where classes of all grades of proficiency have made it their principal text book, has impressed the author of this Key very favorably in regard to the superiority of that arithmetic as a manual for learners. This experience, and valuable suggestions from others, have been his guide in the preparation of the Key. Wherever explanations and illustrations have been deemed necessary, they have been given as fully as was thought proper, regard always being had to the reasonable limits desirable in a work of this kind. In several instances operations have been performed, or their results indicated, to save the teacher the drudgery of what would be to him mere mechanical labor — a service which the author believes teachers will fully appreciate.

Although the Key has been prepared partly at the suggestion of Mr. Chase, the author of the Arithmetic, and with his approbation, it is proper to state here that he should not be held responsible for any portion of it. And it is equally proper that acknowledgments should here be made, as they most cheerfully are, for his kindness and assistance in examining and correcting the sheets as they have come from the press.

Should it be found that the Key approximates, even, to the Arithmetic, in point of simplicity, completeness, and adaptedness to its purpose, its merits will equal the expectation of the author.

MILLBURY ACADEMY, Mass.,
December 16, 1852.

SUGGESTIONS TO TEACHERS AND OTHERS.

THE favor with which the Common School Arithmetic has been received, by those qualified to judge of its merits, fully entitles it, as its friends believe, to a candid examination by Teachers, School Committees, and others interested in education, and especially in the department of mathematics. The work has been examined by some of the best teachers of the day, and has received their highest commendation; and especially has it stood well the test of the school room; a trial without which no text book can present its claims for general introduction or use. In several instances where it has been taken up with seeming objections, and in some cases with positive prejudices, against it, a further acquaintance with its character, and a trial of it in the school room, have entirely removed those objections and prejudices, and won for it almost unqualified praise. The testimony of well-informed professional teachers, who have given it a fair examination and trial, has always been in its favor.

Some of the essential features of the work, in which it differs, more or less, from most arithmetics, may here be briefly alluded to.

At the commencement of the work, Decimals are introduced with Whole Numbers, and their operations are taught in connection. This seems much more philosophical than to reserve the consideration of Decimals, to be taken up with Fractions after the pupil has learned all the fundamental rules of operation in Whole Numbers. The numeration and notation of Whole Numbers and of Decimals are so similar, that there can be no good reason for separating them; and they have, also, such an intimate relation to each other, that it seems incomplete to teach one without the other. No more time is necessary to teach them both in connection than to teach one without the other.

The examples in Numeration and Notation, and in the other fundamental rules, are specially designed to give the pupil a clear understanding of the first principles of the science and of their application, and to prepare him, by an easy progress, for the more difficult portions.

Some cases have come to the knowledge of the author where pupils, who had gone over a considerable portion of another text book, were found so radically deficient in the first principles of numbers, that their teachers compelled them, very much against their will, to spend several weeks on the introductory chapters of Chase's Arithmetic. The result was an understanding of numbers, and a degree of facility in operating them, as gratifying to the pupils themselves as to others, and which the former exhibited with no small degree of pride and satisfaction.

In Fractions, the practice of giving a great number of rules for the various applications of the few principles involved in that section (which applications differ but little from each other) is here discarded, and the operations are confined to the elementary principles of Fractions, while nothing essential to a good understanding of the subject has been omitted. In § 75, a decided improvement has been made in the rule for the Reduction of Fractions to a Common Denominator. The rules for Interest and Banking, for the Reduction of Sterling Money, for the Extraction of the Roots, and for Mensuration, are recommended for special examination. They are easy of explanation, are based upon the most obvious principles of those rules, and are eminently calculated for practical use in business computations.

Considerable prominence is given to the subject of Analysis, for the evident reason that its applications may be extended to almost all operations in business, and also because its use affords one of the most thorough exercises for the discipline of the pupil's mind. It is the application of this rule that constitutes the distinguishing feature of that deservedly popular work, Colburn's Intellectual Arithmetic.

The method of operating and explaining Alligation Alternate may also be cited as deserving notice. The old method of performing the examples by "connecting the ingredients," &c., — a very questionable and unsatisfactory method, — is here superseded by an easy modification of the rule for Average.

Throughout the work, the rules and explanations are expressed in unusually clear and intelligible language, and are concise and free from that tedious prolixity which often compels the pupil to throw aside his book in actual despair. No arithmetic, in common use, has the principle of cancelling so generally applied as this work. It is claimed that this feature is one of no small merit.

The number of examples in the book is unusually large; and this is specially designed to meet a desideratum in text books that have preceded it. It is a great injury to a pupil, in any branch of study, to be hurried from one subject to another, without dwelling upon any one thing sufficiently to acquire a clear understanding of it in its various relations. It effectually prevents all thoroughness in teaching, and induces habits of mind that are wholly unfitted for vigorous progress, and that will ultimately end in an entire inability to grasp and master any important question. The object of study is but half attained, and superficial scholars are the result. Particularly is this true in arithmetic, where there is need of *more practice.* At the commencement of a new rule, most pupils are obliged to perform several examples before they fully comprehend the principle to be inculcated. But after this, the operation of a considerable number of additional sums will secure what has been already acquired, and give increased facility and readiness in performing operations, proving true the old maxim, that "practice makes perfect." A rule thus acquired is seldom forgotten.

Those who object to a large number of examples must do it upon some other ground than the want of time to perform them; for it is a well-known fact, which no experienced teacher will presume to gainsay, that when a pupil enters upon a new section in arithmetic, and is required to perform, say, for example, twenty sums, more time is necessary to learn the rule and to work the first five examples than will be required to perform the remaining fifteen. This view of the subject is worthy of consideration.

Other portions and features of the work might, with equal propriety, be selected for special notice; but the necessary limits of this introduction preclude the possibility of so doing.

PARTICULAR REMARKS.

☞ In the Key, the operations of examples are, to a considerable extent, indicated by means of signs, instead of being performed at length, for the purpose of bringing the work within reasonable limits. It is presumed that no one, who will have occasion to use the Key, will be unable to understand the usual algebraic signs there used.

In Federal Money, the answers are usually given to the nearest cent; (see Arithmetic, page 133, note;) and in Sterling Money, to the nearest farthing. And in many cases, also, where computations in money are not concerned, a fraction exceeding *one half* is reckoned as a unit, where such would be the rule in practical business.

The sign + following a decimal, shows that the answer is not exact, and that the decimal may be continued further, if necessary.

The answers to a very few examples have been omitted, where they are evident at a glance of the eye.

In operating examples, the method of explanation adopted has usually been that which was considered best adapted to illustrate the rule given in the Arithmetic to which this Key is a companion. Teachers will, of course, vary this method, for the purpose of more particular illustration, and to give greater range to the pupil's knowledge of the subject.

NUMERATION TABLE.

The value of the number given below, consisting of sixty-six figures, is expressed in words as follows: Four hundred and twenty-three vigintillions, four hundred and eighty-six novemdecillions, seven hundred and ninety-four octodecillions, two hundred and thirty-five septendecillions, three hundred and seventy-six sexdecillions, eight hundred and ninety-one quindecillions, three hundred and fifty-two quatuordecillions, six hundred and ninety-eight tredecillions, two hundred and four duodecillions, three hundred and forty-five undecillions, seven hundred and eighty-nine decillions, two hundred and sixty-eight nonillions, one hundred and seventy-three octillions, five hundred and seventy-one septillions, two hundred and eight sextillions, nine hundred and ninety-nine quintillions, two hundred and thirty-two quadrillions, five hundred and seventeen trillions, three hundred and three billions, one hundred and twenty-six millions, seven hundred and ninety-eight thousands, three hundred and twenty-one.

Vigintillions.	Novemdecillions.	Octodecillions.	Septendecillions.	Sexdecillions.	Quindecillions.	Quatuordecillions.	Tredecillions.	Duodecillions.	Undecillions.	Decillions.	Nonillions.	Octillions.	Septillions.	Sextillions.	Quintillions.	Quadrillions	Trillions.	Billions.	Millions.	Thousands.	Units.
423,	486,	794,	235,	376,	891,	352,	698,	204,	345,	789,	268,	173,	571,	208,	999,	232,	517,	303,	126,	798,	321.

KEY.

I. NUMERATION.

¶ 13. 101–118. (pp. 16–23.) 4 thous. 516; 60 thous. & 60; 200 thous. & 2; 15 thous. 384; 17 thous. & 9; 284 thous. 391; 100 thous. 901; 72 thous. & 53; 40 thous. & 80; 900 thous. & 900; 33 thous. & 15; 284 thous. & 7; 197 thous. & 40; 30 thous. 603; 804 thous. & 7; 1 mill. 15 thous. & 2; 19 mill. & 70; 40 thous. & 40.

119–130. 40 mill. 117 thous. & 20; 8 mill. 40 thous. & 8; 100 mill. & 900; 1 thous. & 9; 3 mill. 807 thous.; 500 mill.; 400 mill. & 16; 7 bill. 3 mill. 605; ten; 28 mill. 400 thous. 127; 4 trill. 15 bill. 90 mill. 600 thous.; 7 quad. 200 bill. 50 thous. & 7; 19; 5 trill. 649 bill. 380 mill. 75 thous. 109; 50 quin. 34 quad. 700 bill.

¶ 17. 181–190. (p. 18.) 120 mill. 7 thous. & 95; 12 mill. 709 & 5 tenths; 1 mill. 200 thous. & 70, & 95 hundredths; 120 thous. & 7, & 95 thousandths; 12 thous., & 7 thous. & 95 ten-thousandths; 1 thous. 200, & 7 thous. & 95 hundred-thousandths; 120, & 7 thous. & 95 millionths; 12, & 7 thous. & 95 ten-millionths; 1, & 20 mill. 7 thous. & 95 hundred-millionths; 120 mill. 7 thous. 95 billionths.

¶ 19. 211–222. (p. 19.) 1003; 2017; 5020; 11400; 40180; 200001; 5000000; 900000000; 2000002; 90060004; 200000060; 807009000.

223–233. 8000000000; 2000007015; 10000000020; 1070000000; 200000041000; 1000001010; 7000000000000; 60003000000000; 2000000200000; 508000028000050; 1010000015000.

234–240. 1000000000000100; 2000040000; 6000000000000000006; 7000059; 3004; 200000010; 10000030209000000005000.

¶ **21.** **256–268.** (p. 20.) .004; 1.016; 9.0007; .0095; 114.02; 200090; 20009; 2000.9; 200.09; 20.009; 2.0009; .20009; .020009.

269–274. 704.001005; .0013000063; .06 & .003; .02 & .007; .04 & .005; .6 & .03; .2 & .07; .4 & .05; 6 & .3; 2 & .7; 4 & .5; 1 & .9; 6 t. & 3 u.; 1 t. & 7 u.; 9 t. & 4 u.; 3 t. & 7 u.

275–280. 6 hund. & 3 tens., 1 h. & 4 t., 8 h. & 7 t., 2 h. & 2 t.; 6 thous. & 3 hund., 4th. & 9 h., 27 th. & 5 h.; 50, 500, 5000, 50000; 150, 1500, 15000, 150000, 1500000; 280, 2800, 28000, 280000, 2800000, 28000000; 0.2407, 2.407, 24.07, 240.7, 2407, 24070, 240700.

¶ **23.** **341–345.** (p. 23.) $14000003.0047; $1.65; $16500; $28001.005; $105026.00206.

346–350. $1650000; .0165; $5005005.000005; $35019.01001; $23000060001.0000408.

II. ADDITION.

¶ **25.** **92–100.** (p. 26.) 100; 93; 94; 85; 91; 98; 95; 88; 82.

¶ **27.** **101–105.** (p. 27.) 99; 118; 666; 799; 1099.

106–110. (p. 28.) 148; 148; 541; 761; 1652.

111–115. 8155; 10719; 6797; 98509; 61501.

116–120. 176180; 63676; 24360; 19875; 243962.

121–125. 2980; 3751; 15914; 2152; 2516.

221–225. (p. 29.) 769.83; 7.1734; 1134.579; 32.15; 103.57.

226–230. 523.877; 7.3699; .01231; 3402.55; 201.4753.

231–240. 52.8296; 2.048; 1000; 7723.05; 1429.495; $451.05; $977; $1100; $108.412; $529.862.

¶ 28. 241–250. A. D. 1799; 2091; $852; 440091; 1190160; 446 m.; 2300 cts.; $15500; 366 dy.; 156.

251–260. 384 a.; $5856; $29.375; $27.75; $10022.875; 189.375 yd.; 901 bbl. for $4206.625; $26159; $29.875; 10 pieces, 252.25 yds., $1185.125.

III. SUBTRACTION.

¶ 30. 81–90. (p. 34.) 171; 565; 783; 1121; 70; 1000; 3040; 8632; 1350; 80404.

91–100. 3; 206; 332; 460; 35; 93200; 7000; 6060; 4101; 7674.

¶ 32. 103–110. (p. 36.) 25; 26; 45; 85; 7; 38; 87; 36.

111–120. 273; 228; 111; 60; 147; 192; 291; 289; 16; 163.

121–130. 84; 538; 606; 195; 868; 389; 681; 6; 367; 385.

131–140. 7774; 7149; 6998; 7421; 95; 1682; 7499; 6016; 6294; 2147.

141–150. 612; 1903; 3897; 692; 8008; 1601; 438; 794; 296; 8086.

151–160. 389; 899; 118; 1866; 19; 8642; 9713; 10999; 9999; 7097.

161–170. 113889; 114929; 103080; 31247; 9033; 99999; 99034; 79936; 17728; 68612.

171–180. 839; 95; 3087; 1925; 23993; 3739; 2284; 3986; 8094; 1898.

181–190. 56.492; 297.09; .00988; 3608.9; 1.6248; .48077; 50.09; 48.902; 97.62; .6961.

191–200. 8.036; .06012; .0375; $12.625; 142.75; $0.81; $9.034; $5.11; 4.929; $27.182.

201–205. $9.37; $29.50; 194.06; 12.3456; $26.625.

206–215. $980.375; $69; $620.90; $58.125; $969.67; $528.95; $932.991; $997.94375; $999.999; $166.975.

391–400. 50746.5 ; 64638 ; 370440 ; 4488.75 ; 79.4745 ; 649.4985 ; 52825.5 ; 7069545 ; 1936.305 ; 33660.9.

¶ 42. 401–410. (p. 49.) 5457 ; 4173 ; 6099 ; 7383 ; 9309 ; 9951 ; 11877 ; 15087 ; 18939 ; 19581.

411–420. 79871 ; 111518 ; 58773 ; 123574 ; 91927 ; 135630 ; 131109 ; 143165 ; 58773 ; 167277.

421–430. 487426 ; 979678 ; 2446782 ; 4362704 ; 2934208 ; 593598 ; 3112770 ; 4242054 ; 1930400 ; 3387852.

431–440. 2258520 ; 3961980 ; 6775560 ; 5445330 ; 9168060 ; 4134240 ; 7703850 ; 8507730 ; 3416490 ; 9560430.

441–450. 54752079 ; 31513712 ; 22534401 ; 65566195 ; 8305301 ; 20647173 ; 28458200 ; 74800132 ; 57485564 ; 32914155.

451–460. 30664280000 ; 332209240000 ; 368048600000 ; 3095006800000 ; 343718000000 ; 293627860000 ; 196583524000 ; 309303718000 ; 10574156000 ; 3858138000000.

461–470. 47237785920 ; 36609168960 ; 24090091200 ; 44317195200 ; 5830790400 ; 34861939200 ; 16247512800 ; 7288842240 ; 41380545600 ; 394859520000.

¶ 44. 521–530. (p. 51.) .162 ; .0392 ; .6084 ; .008649 ; 75.739 ; 169.88 ; 1581 ; .1505 ; 5.544 ; 49.

531–540. 1.5246 ; .03585 ; .06183 ; .032364 ; .0018438 ; .00217856 ; .000084568 ; .1029 ; .001296 ; .93304.

541–550. $1.677 ; $0.41925 ; $2.09625 ; $1.25775 ; $5.031 ; $6.708 ; $1.048125 ; $0.005240625 ; $26.3289 ; $49.27585.

551–560. 4761 ; 37.41 ; $5.625 ; $2.856 ; $7.453125 ; $3.90625 ; 41400 ; .00079898 ; $2967.30 ; $0.003515625.

561–570. $0.4662 ; $22.31 ; $91.806 ; $8.722 ; $35.9375 ; $456.484 ; $134.995 ; $0.335353 ; 48790.44 ; 135.1224.

¶ 45. 571–586. 4459 ; 156585 ; 233600 ; 256,125440 ; 525600 ; 42560 ; 336 ; 1813 dy. ; 3441 ; 19,323 ; 888 ; 254198 ; 64428 times ; 335699 ct. ; 1219420866400 ; $410.625.

587.

$$\begin{array}{rl} 5 \times \$4.75 &= \$23.75 \\ 7 \times \$1.125 &= 7.875 \\ 9 \times \$0.875 &= \underline{7.875} \\ & \$39.50 \text{ Ans.} \end{array}$$

588.

$$\begin{array}{rl} 19 \times \$3.75 &= \$71.25 \\ 14 \times \$5.50 &= 77.00 \\ 29 \times \$1.56 &= 45.24 \\ 145 \times .67 &= \underline{97.15} \\ & \$290.64 \text{ Ans.} \end{array}$$

589.

$$\begin{array}{rl} 5 \times \$2.625 &= \$13.125 \\ 16 \times \$1.375 &= 22.000 \\ 68 \times \$1.25 &= 85.000 \\ 60 \times .375 &= \underline{22.500} \\ & \$142.625 \text{ Ans.} \end{array}$$

590.

$$\begin{array}{rl} 39 \times \$16.75 &= \$653.25 \\ 163 \times .08 &= 13.04 \\ 95 \times .34 &= 32.30 \\ 348 \times .115 &= \underline{40.02} \\ & \$738.61 \text{ Ans.} \end{array}$$

591–599. 2610 ; $2979.20 ; 17.04, — 1.1718 ; $163.5875 ; $37.734375 ; $43.134 ; $42.2875 ; 180, — 3, — 7.008, — 21 ; $396.68.

600.

$$\begin{array}{rl} 4 \times 31.5 \times .125 &= \$15.75 \\ 6 \times 30 \times .115 &= 20.70 \\ 5 \times 31 \times .11 &= 17.05 \\ 7 \times 29 \times .13 &= \underline{26.39} \\ & \$79.89 \text{ Ans.} \end{array}$$

MISCELLANEOUS EXAMPLES

IN ADDITION, SUBTRACTION, AND MULTIPLICATION.

¶ **46.** **601–610.** (p. 54.) 262 ; 26280 ; 700, — 993 ; 3669 ; 237 ; 123 oz., — 1977 dr. ; 2637 min., — 158220 sec. ; 1209 ; 10092225 ; 1782862942.

611–620. 384 ; 11919 lb. ; $23.92 ; $18.90 ; $899 ; 37 m. ; 5206916 ; 3217 gall. ; 70297 ; 63998.

621–629. 792 ; 447066 ; 293880 ; 20377 ; 583 ; $231.25 ; $894.375 ; $82.25 ; $2719.

630. 56 + 91 + 100 = 247 kegs. 421 — 247 = 174 kegs.
56 × $12.50 = $700.
91 × 11.75 = $1069.25.
100 × 12.25 = $1225.
174 × 11.50 = $2001.
$4995.25 Ans.

631–639. 1st. share $125, 2d. $375, 3d. $500, 4th. $1000, whole estate, $2000; $73.9375; $136.25; I lose $9.10; $414.4375; $340.20; $1021.875; 3459600; 356400829.

640–645. 1330.90713856; 381078.125; 537824; 47045881; .00001801088541; 94.206436.

646–649. 3486784401; 1000000000000000; .0601692057; 1.41158161.

650. 3d power of 10 = 1000. 4th power of 17 = 83521. 1000 + 83521 = 84521. 5th power of 10 = 100000. 3d power of 46 = 97336. 100000 — 97336 = 2664. 84521 × 2664 = 225163944.

651. 832884 lb.

V. DIVISION.

¶ **48.** **141–150.** (p. 60.) 2134; 4020; 3344; 5402; 731; 823; 902; 722; 713; 9004.

151–160. 1221; 3210; 2033; 411; 703; 531; 933; 623; 802; 60132.

161–170. 1001; 2002; 702; 612; 321; 411; 522; 910; 601; 812.

171–180. 1011; 211; 410; 601; 310; 700; 511; 800; 901.

181–190. 211; 301; 604; 507; 810; 906; 1007; 205; 808; 906.

191–200. 301; 211; 402; 105; 609; 810; 704; 908; 511; 802.

201–210. 211; 401; 103; 503; 907; 804; 109; 710; 408; 1006.

211–220. 302 ; 107 ; 410 ; 611 ; 205 ; 709 ; 108 ; 1107 ; 506 ; 804.

¶ 50. 221–230. (p. 61.) 854 ; 4057 ; 3609 ; 678 ; 4875 ; 1687 ; 2968 ; 1395 ; 2269 ; 4989.

231–240. 753 ; 475 ; 2379 ; 1378 ; 1674 ; 2775 ; 2339 ; 744 ; 2936 ; 985.

241–250. 259 ; 1277 ; 1756 ; 2333 ; 685 ; 2499 ; 2192 ; 1358 ; 864 ; 1973.

251–260. 358 ; 1643 ; 1889 ; 756 ; 3864 ; 937 ; 6541 ; 17791 ; 1974 ; 1359.

261–270. 1454 + 5 ; 239 + 1 ; 983 + 3 ; 7013 + 2 ; 12253 + 1 ; 7458 + 2 ; 11795 ; 14958 + 2 ; 1666 + 3 ; 7187 + 3.

271–280. 747 + 6 ; 1227 + 1 ; 1394 + 5 ; 617 + 6 ; 1712 + 6 ; 3683 + 5 ; 2642 ; 969 + 6 ; 863 + 3 ; 7581 + 5.

281–290. 3138 + 5 ; 7875 ; 1125 + 7 ; 465 + 5 ; 653 + 3 ; 582 + 3 ; 886 + 2 ; 9149 + 3 ; 4166 + 5 ; 8838 + 5.

291–300. 274 + 2 ; 1261 + 1 ; 972 + 1 ; 2177 + 6 ; 9333 + 3 ; 9739 + 3 ; 3566 + 7 ; 5417 + 4 ; 676 + 3 ; 779 + 4.

¶ 52. 301–320. (p. 64.) 43520 ; $29013\frac{1}{3}$; 21760 ; 17408 ; $14506\frac{4}{6}$; $12434\frac{2}{7}$; 10880 ; $9671\frac{1}{9}$; 8704 ; $7912\frac{8}{11}$; $7253\frac{4}{12}$; $6695\frac{5}{13}$; $6217\frac{2}{14}$; $5802\frac{10}{15}$; 5440 ; 5120 ; $4835\frac{10}{18}$; $4581\frac{1}{19}$; 4352 ; $4144\frac{16}{21}$.

321–330. $8822\frac{1}{22}$; $8438\frac{11}{23}$; $8086\frac{21}{24}$; $7763\frac{10}{25}$; $7464\frac{21}{26}$; $7188\frac{9}{27}$; $6931\frac{17}{28}$; $6692\frac{17}{29}$; $6469\frac{15}{30}$; $6260\frac{25}{31}$.

331–340. $9941\frac{69}{91}$; $9833\frac{64}{92}$; $9727\frac{89}{93}$; $9624\frac{44}{94}$; $9523\frac{15}{95}$; $9423\frac{92}{96}$; $9326\frac{78}{97}$; $9231\frac{62}{98}$; $9138\frac{38}{99}$; 9047.

341–350. 9944 ; $9745\frac{6}{50}$; $9554\frac{2}{51}$; $9370\frac{16}{52}$; $9193\frac{27}{53}$; $9023\frac{14}{54}$; $8859\frac{11}{55}$; 8701 ; $8548\frac{20}{57}$; $8400\frac{56}{58}$.

351–360. $1013\frac{40}{360}$; $1010\frac{110}{361}$; $1007\frac{186}{362}$; $1004\frac{268}{363}$; $1001\frac{356}{364}$; $999\frac{85}{365}$; $996\frac{184}{366}$; $993\frac{289}{367}$; $991\frac{32}{368}$; $988\frac{148}{369}$.

361–365. $1563\frac{139}{158}$; $664\frac{85}{372}$; $405\frac{448}{609}$; $1016\frac{205}{243}$; $1000\frac{93}{247}$.

366–370. $10022\frac{130}{159}$; $1675\frac{703}{951}$; $2696\frac{292}{591}$; $1846\frac{530}{863}$; $3706\frac{48}{430}$.

371–375. $50615\frac{501}{1640}$; $22422\frac{2857}{3702}$; $10004\frac{5213}{8297}$; $11019\frac{2274}{7533}$; $20705\frac{2756}{4009}$.

376–380. $19092\frac{1720}{2931}$; $9978\frac{3748}{5608}$; $10021\frac{3108}{5584}$; $5972\frac{2732}{9370}$; $6865\frac{3757}{8151}$.

381–385. $59587\frac{6}{1190}$; $16163\frac{1455}{4387}$; $8403\frac{4022}{8438}$; $7628\frac{6276}{9295}$; $9409\frac{2312}{7536}$.

386–390. $9998\frac{2279}{6701}$; $7968\frac{3933}{8408}$; $8421\frac{1401}{7956}$; $15476\frac{3273}{4329}$; $7091\frac{3109}{9448}$.

391–395. $7496\frac{3944}{4008}$; $15299\frac{676}{1964}$; $10019\frac{931}{2999}$; $3428\frac{1492}{8765}$; $3226\frac{7400}{9312}$.

396–400. $4023\frac{33569}{180327}$; $7747\frac{426338}{527590}$; $9362\frac{7475804}{9998764}$, $539792\frac{2443392}{2631972}$; $175122\frac{32800187}{49078323}$.

¶ 54. 401–410. (p. 66.) 18; 45; 225; 300; 150; 500; 125; 25; 600; 240.

411–420. 315; 1260; 1575; 2100; 9000; 70000; 5625; 4375; 50000; 840.

421–430. 35; 14; 55; 220; 50; 275; 87.5; 437.5; 60.15625; 343.75.

431–440. 48.5; 242.5; 1.2125; 60.625; 3.03125: 7.76; 1.552; $51\frac{1}{18}$; $570\frac{10}{17}$; 6.339 +.

441–450. .007; .0003; .000042; .00168; .000065625; .13125; .0002625; .0056; .000024; .0000125.

451–460. \$60; \$3; \$0.008; \$0.075; \$0.50; .5; 3.75; \$0.0125; \$0.000279 +; .0625.

461–470. \$0.11; 1.1; .55; .05; \$0.015625; \$0.78125; \$195.3125; 550; \$0.000859375; .000694+.

¶ 55. 521–530. (p. 67.) 27 m.; 475 cts.; 190; 125; 29 dy.; 407; 875; 193; 37 bu.; 361.

531–540. 978; 598 + 15; 1097; 1930; 19762 p. 3 f.; 46507 s. 9 p.; 22178 pd. 7 s.; 1183145 p., — 98595 s., — 4929 pd.; 14748 min., — 245 h., — 10 dy.; 47000 ft., — 15666 yd.

541–550. 87 dy.; 297; $41\frac{132}{365}$; 3270; 549; 698; 80891; 329 pd. 10 s. 4 p.; \$1.75; 79.

551–562. \$439.19*; 194; \$1.125; \$0.94; 27 dy.; \$1.05; \$1.09; 37; $265\frac{14}{25}$; \$0.125; $61\frac{1}{3}$ gall.; \$1.61.*

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* *Nearly.* See INTRODUCTION, "Particular Remarks."

¶ **56.** **563–575.** (p. 69.) 738$\frac{26}{113}$; 3963; 17; 52560; 604800; \$37.50; 47; 1811$\frac{3}{12}$; \$1194.80; \$187; 4 and 76; 984 cents lost; \$208.

576–586. \$17; 774; 2 cts.; 39; 218; 619; 57; 1825; 94; 1369; \$85.

587. \$240 — \$45 = \$195. \$195 ÷ 5 = 39 yds. Ans.

588. \$195 — \$45 = \$150. \$150 ÷ 39 = \$3$\frac{33}{39}$ Ans.

589. 13 + 24 = 37. 703 ÷ 37 = 19 Ans.

590. 2 apples to each girl, and 1 to each of two boys, ("twice as many") = 4 apples. 108 ÷ 4 = 27 girls. 27 × 2 = 54 boys.

591–605. \$23; 56; 50 weeks; \$19.375; \$0.125; 4 pd. 10 s. 5 p.; 588; 79; 5850; 35 cts.; 80 cts.; \$26.25; 32 lb.; 56 lb.; 15 w.; 5 w.

606–616. 9 dy.; 3 w.; 160 w.; \$726.75; \$45.75; \$128.50; \$968.50; \$925.45; \$70.20; \$18; 7.

617–623. 20; 49 dy.; 50 bu.; \$48; 60225 lb.; \$18.50; \$0.25.

624.

$$\begin{array}{rrr} 56 \times & \$12.50 = & \$\ 700.00. \\ 91 \times & 11.75 = & 1069.25. \\ \underline{100} \times & 12.25 = & \underline{1225.00.} \\ \text{247 kegs cost} & & \$2994.25. \end{array}$$

\$4995.25 — \$2994.25 = \$2001. 421 — 247 = 174 kegs. 2001 ÷ 174 = \$11.50 Ans.

625. .29 + .31 + .15 = .75. \$1137.50 × .75 = \$853.125.

626. 8th power of 6 = 1679616. 7th power of 3 = 2187. 1679616 ÷ 2187 = 768 Ans.

627–650. .5; .25; .5; .75; .2; .4; .6; .8; .5; .333 +; .375; .4375; .28125; .44; .666 +; .6363 +; .888 +; .0101 +; .515625; .056; .0703125; .714285 +; .9090 +; .1538 +.

651–675. 14; 18; 8; 17$\frac{8}{13}$; 7$\frac{8}{11}$; .8333 +; .888 +; 19.4; 4.625; .3636 +; .5833 +; 2.666 +; 9.71428 +; 28.666 +; .875; 5.9375; .6875; 4.75; .2105 +; 9.588 +; 3$\frac{8}{43}$; .5625; 274$\frac{2}{19}$; 684; 3716.

PRIME AND COMPOSITE NUMBERS.

¶ 59. (p. 79.)

1.	$2\underline{)60}$	$2 \times 2 \times 3 \times 5$	
	$2\underline{)30}$	or $2^2 \times 3 \times 5$ Ans.	
	$3\underline{)15}$		
	$5\underline{)5}$		
	1		
2.	$2\underline{)32}$	$2 \times 2 \times 2 \times 2 \times 2$	
	$2\underline{)16}$	or 2^5 Ans.	
	$2\underline{)8}$		
	$2\underline{)4}$		
	$2\underline{)2}$		
	1		

3–20. 2×3^3; $2 \times 3 \times 7$; $3^2 \times 7$; $2^2 \times 3^2$; $2^3 \times 7$; $3^3 \times 5$; 3^4; 7×13; 2×19; $2^2 \times 11$; $2 \times 3 \times 11$; $2^3 \times 3^2$; 2^6; $2^4 \times 3$; 5×19; $2 \times 3^2 \times 5$; $2^3 \times 11$; $2^2 \times 23$.

21–40. $2 \times 3 \times 17$; $2 \times 3 \times 5 \times 17$; $2^4 \times 3 \times 17$; $2^2 \times 7 \times 11$; $2^3 \times 3 \times 17$; $3^3 \times 19$; 7^3; $2 \times 3^2 \times 23$; $2^3 \times 89$; $2^5 \times 29$; $2^3 \times 3^4$; $3^3 \times 37$; $2^6 \times 3^2$; $2^2 \times 3 \times 47$; 17^2; $2 \times 3 \times 7^2$; $2^2 \times 193$; $2 \times 7 \times 11$; $2^3 \times 23$; $3 \times 17 \times 19$.

41–60. $2 \times 13 \times 23$; $2^2 \times 3 \times 23$; $2 \times 3 \times 7 \times 11$; $3 \times 13 \times 17$; $2 \times 13 \times 37$; $2^3 \times 3 \times 5 \times 7$; $2 \times 3 \times 137$; $2^2 \times 3 \times 7 \times 11$; 13×19; $2^2 \times 3^3 \times 5$; $2 \times 3 \times 5 \times 17$; $3 \times 5 \times 61$; $7 \times 11 \times 13$; $2^3 \times 3 \times 7 \times 11$; $2 \times 7^2 \times 19$; $2^2 \times 5 \times 59$; $2^2 \times 3^2 \times 37$; $2^5 \times 3 \times 17$; $2^3 \times 13 \times 19$; $2^2 \times 3^5 \times 7$.

¶ 61. **61–80.** (p. 80.) 227; 229; 233; 239; 241; 251; 257; 263; 269; 271; 277; 281; 283; 293; 307; 311; 313; 317; 331; 337.

81–90. 347; 349; 353; 401; 409; 439; 953.

91–100. 359×1; $2^2 \times 7 \times 13$; 367×1; 373×1; $2 \times 3^3 \times 7$; 379×1; 383×1; $2 \times 7 \times 191$; $2 \times 3^2 \times 241$; $2 \times 11 \times 13 \times 31$.

LEAST COMMON MULTIPLE.

¶ 64. **101–110.** (p. 82.) 6 ; 12 ; 30 ; 30 ; 42 ; 42 ; 24 ; 36 ; 105 ; 120.

111–120. 60 ; 72 ; 84 ; 54 ; 98 ; 245 ; 231 ; 297 ; 75 ; 385.

121–125. 72 ; 36 ; 616 ; 840 ; 2145.

126–130. 120 ; 1980 ; 3276 ; 3059 ; 180.

131–135. 96 ; 864 ; 420 ; 2052 ; 660.

136–140. 2520 ; 1386 ; 72 ; 44100 ; 210.

GREATEST COMMON DIVISOR.

¶ 67. (p. 84.)

141. 12)12 — 14.
0 — 2)2 — 12.
0 — 0 2 Ans.

142–150. 3 ; 13 ; 4 ; 5 ; 9 ; 27 ; 2 ; 7 ; 6.

151. 4)4 — 6 — 10
0 2)2 2 4 2 Ans.

152–160. 3 ; 5 ; 7 ; 6 ; 10 ; 14 ; 15 ; 12 ; 21.

161. 7)7 — 8 — 9 — 12
1) 1 — 2 — 5 — 7 1 Ans.

162–170. 4 ; 9 ; 6 ; 3 ; 14 ; 16 ; 24 ; 42 ; 11.

171–180. 12 ; 77 ; 81 ; 1 ; 32 ; 107 ; 327 ; 716 ; 96 ; 1.

181–190. 2 ; 46 ; 25 ; 125 ; 233 ; 74 ; 48 ; 58 ; 106 ; 421.

191–195. 107 ; 647 ; 421 ; 1 ; 409.

196–200. 1 ; 1 ; 449 ; 323 ; 339.

¶ **69.*** **201–210.** (p. 86.) 2; $\frac{4}{3}$; 7; 7; $\frac{4}{3}$; $\frac{3}{4}$; $\frac{7}{5}$; 2 : 5; 5 : 1; 4 : 7.

211–220. $\frac{5}{9}$; 2; $\frac{11}{10}$; $\frac{7}{5}$; $\frac{1}{3}$; $\frac{2}{7}$; 7 : 5; 1; 1; $\frac{1}{3}$.

221–230. 3 ÷ 2; 3 : 5; $\frac{13}{5}$; $\frac{3}{7}$; 3 : 2; 2 ÷ 5; $\frac{2}{9}$; $\frac{14}{3}$; 30; $\frac{1}{40}$.

231–240. $\frac{9}{8}$; $\frac{9}{11}$; 7 : 5; $\frac{2}{5}$; $\frac{1}{2}$; $\frac{4}{3}$; $\frac{3}{11}$; $\frac{1}{2}$; $\frac{7}{19}$; $\frac{11}{40}$.

241–250. $\frac{8}{21}$; $\frac{1}{105}$; 273; $\frac{5}{3}$; $\frac{15}{7}$; $\frac{4}{3}$; $\frac{3}{5}$; $\frac{2}{3}$; $\frac{13}{3}$; $\frac{4}{3}$.

251–265. Let these examples be performed mentally, for the purpose of testing the pupil's knowledge of the subject.

266–270. 3948; 12900; 21420; 1872; 49049.

271–275. 806; 1496; 44; 26; 366.

276–295. 72; 48; 144; 216; 96; 108; 27; 54; 48; 32; 288; 432; 18; 16; 12; 9; 8; 6; 4; 3.

296–315. 144; 36; 54; 216; 162; 432; 108; 81; 48; 24; 27; 12; 9; 6; 16; 8; 3; 4; 324; 648.

316–340. 20; 80; 84; 21; 168; 240; 280; 210; 7; 6; 3; 30; 60; 120; 15; 14; 28; 56; 112; 8; 16; 35; 24; 48; 12.

341–360. 270; 405; 486; 162; 90; 135; 810; 45; 54; 30; 81; 15; 10; 5; 9; 27; 243; 18; 6; 3.

361–390. 72; 150; 300; 450; 600; 120; 360; 225; 12; 6; 8; 4; 2; 3; 9; 18; 36; 180; 90; 45; 15; 30; 60; 20; 5; 50; 10; 100; 200; 25.

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[*] Teachers ought to insist that pupils are thoroughly acquainted with the principles and practice of cancelling before they are allowed to go beyond ¶ **69.** It is the custom of many teachers to disregard the subject, or to call the attention of learners to it but slightly. This is a great loss to the pupil; for cancelling greatly abbreviates most mathematical operations, and the principle upon which it is performed (not a difficult one) is an important one for the pupil to understand at this stage of his progress.

These remarks may be applied with equal propriety to ¶ **57** to ¶ **67**, inclusive.

VII. FRACTIONS.

CASE I.

¶ 71. 1–20. (p. 91.) 2; 3; 5; 2; 6; 4; 3; 2; 4; 3; 9; 6; 3; 2; 7; 3; 7; 5; 11; 3.

21–40. $4\frac{1}{2}$; $2\frac{3}{4}$; $3\frac{2}{3}$; $2\frac{1}{5}$; $2\frac{2}{7}$; $2\frac{6}{8}$; $3\frac{4}{9}$; $4\frac{7}{10}$; $8\frac{10}{11}$; $8\frac{4}{12}$; $6\frac{7}{13}$; $7\frac{13}{14}$; $8\frac{9}{15}$; $30\frac{7}{16}$; $19\frac{6}{17}$; $28\frac{15}{18}$; $44\frac{6}{19}$; $17\frac{13}{20}$; $19\frac{3}{31}$; $14\frac{17}{45}$.

41–60. .5; .25; .5; .75; .333 +; .666 +; .6; .166 +; .333 +; .5; .666 +; .833 +; .125; .25; .375; .5; .875; .111 +; .222 +; .777 +.

61–80. .333 +; .444 +; .555 +; .666 +; .888 +; $5\frac{2}{5}$; 13; 21; $44\frac{3}{11}$; $10\frac{29}{97}$; $25\frac{1}{35}$; $35\frac{2}{17}$; 20; 23; 10; $43\frac{9}{44}$; $69\frac{65}{93}$; $75\frac{12}{13}$; 48; 38.

81–100. .533 +; $3\frac{2}{3}$; .2727 +; $1\frac{2}{6}$; .75; .625; $19\frac{4}{5}$; $14\frac{18}{19}$; .21875; $96\frac{67}{101}$; 16; .6; .214 +; $19\frac{2}{3}$; $30\frac{8}{29}$; .104; .01066 +; $93\frac{3}{4}$; $9\frac{8}{13}$; $771\frac{28}{83}$.

CASE II.

¶ 72. 101–120. (p. 92.) $\frac{36}{2}$; $\frac{54}{3}$; $\frac{72}{4}$; $\frac{90}{5}$; $\frac{108}{6}$; $\frac{126}{7}$; $\frac{144}{8}$; $\frac{162}{9}$; $\frac{180}{10}$; $\frac{198}{11}$; $\frac{216}{12}$; $\frac{360}{20}$; $\frac{450}{25}$; $\frac{540}{30}$; $\frac{720}{40}$; $\frac{900}{50}$; $\frac{1080}{60}$; $\frac{1350}{75}$; $\frac{1800}{100}$; $\frac{18000}{1000}$.

121–140. $\frac{7}{4}$; $\frac{21}{8}$; $\frac{29}{9}$; $\frac{29}{5}$; $\frac{81}{11}$; $\frac{55}{6}$; $\frac{61}{7}$; $\frac{101}{12}$; $\frac{60}{13}$; $\frac{116}{7}$; $\frac{77}{4}$; $\frac{37}{2}$; $\frac{59}{2}$; $\frac{97}{3}$; $\frac{47}{3}$; $\frac{118}{4}$; $\frac{339}{4}$; $\frac{128}{13}$; $\frac{63}{29}$; $\frac{92}{73}$.

141–160. $\frac{1\cdot3}{10}$; $\frac{13}{100}$; $\frac{130}{1000}$; $\frac{1300}{10000}$; $\frac{1\cdot97}{10}$; $\frac{19\cdot7}{100}$; $\frac{197}{1000}$; $\frac{197000}{1000000}$; $\frac{8}{10}$; $\frac{80}{100}$; $\frac{800}{1000}$; $\frac{800000}{1000000}$; $\frac{215}{10}$; $\frac{2150}{100}$; $\frac{21500}{1000}$; $\frac{21500000}{1000000}$; $\frac{\cdot9}{10}$; $\frac{9}{100}$; $\frac{90}{1000}$; $\frac{90000}{1000000}$.

161–180. $\frac{9}{10}$; $\frac{7}{10}$; $\frac{16}{100}$; $\frac{4}{100}$; $\frac{247}{1000}$; $\frac{13}{1000}$; $\frac{5}{1000}$; $\frac{84}{100}$; $\frac{184}{100}$; $\frac{206}{100}$; $\frac{593}{10}$; $\frac{87}{10}$; $\frac{19}{10000}$; $\frac{802}{10}$; $\frac{802}{100}$; $\frac{3}{10}$; $\frac{62}{100}$; $\frac{119}{100}$; $\frac{4}{100}$; $\frac{83}{1000}$.

181–200. $\frac{31}{12}$; $\frac{87}{100}$; $\frac{71}{17}$; $\frac{91}{11}$; $\frac{4}{37}$; $\frac{84}{1000}$; $\frac{116}{10}$; $\frac{49}{100}$; $\frac{78}{5}$; $\frac{100}{3}$; $\frac{100}{6}$; $\frac{100}{7}$; $\frac{100}{8}$; $\frac{100}{9}$; $\frac{100}{11}$; $\frac{100}{12}$; $\frac{19}{10}$; $\frac{27}{10}$; $\frac{305}{100}$; $\frac{231}{49}$.

CASE III.

¶ **73.** **201–220.** (p. 93.) $\frac{1}{6}$; $\frac{1}{8}$; $\frac{1}{10}$; $\frac{1}{12}$; $\frac{1}{14}$; $\frac{3}{16}$; $\frac{7}{18}$; $\frac{5}{22}$; $\frac{2}{9}$; $\frac{2}{15}$; $\frac{5}{12}$; $\frac{1}{2}$; $\frac{1}{2}$; $\frac{1}{2}$; $\frac{10}{33}$; $\frac{16}{45}$; $\frac{1}{8}$; $\frac{1}{4}$; $\frac{1}{2}$; $\frac{5}{12}$.

221–230. (p. 94.) $\frac{1}{24}$; $\frac{1}{4}$; $\frac{1}{7}$; $\frac{7}{30}$; $\frac{2}{5}$; $\frac{4}{105}$; $\frac{1}{54}$; $\frac{1}{30}$; $\frac{5}{77}$; $\frac{21}{440}$.

231–240. $1\frac{5}{13}$; $\frac{7}{40}$; 1; $\frac{5}{8}$; $\frac{5}{6}$; 1; $\frac{5}{7}$; 1; $\frac{3}{14}$; $\frac{1}{28}$.

241–250. $1\frac{4}{11}$; $3\frac{1}{9}$; $6\frac{3}{7}$; $\frac{20}{143}$; $\frac{1}{8}$; $4\frac{1}{12}$; $\frac{3}{4}$; $1\frac{9}{20}$; $\frac{11}{65}$; $\frac{23}{36}$.

CASE IV.

¶ **74.** **251–270.** (p. 94.) $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{4}$; $\frac{1}{4}$; $\frac{2}{3}$; $\frac{2}{5}$; $\frac{3}{4}$; $\frac{3}{5}$; $\frac{1}{2}$; $\frac{1}{3}$; $\frac{4}{5}$; $\frac{2}{3}$; $\frac{4}{7}$; $\frac{1}{2}$; $\frac{1}{6}$; $\frac{3}{4}$; $\frac{3}{5}$; $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{9}$.

271–290. $\frac{6}{7}$; $\frac{4}{5}$; $\frac{3}{4}$; $\frac{12}{17}$; $\frac{2}{3}$; $\frac{3}{5}$; $\frac{1}{2}$; $\frac{2}{5}$; $\frac{1}{7}$; $\frac{2}{15}$; $\frac{9}{10}$; $\frac{6}{7}$; $\frac{9}{11}$; $\frac{3}{4}$; $\frac{2}{3}$; $\frac{3}{5}$; $\frac{6}{11}$; $\frac{1}{2}$; $\frac{2}{9}$; $\frac{18}{29}$.

291–310. $\frac{9}{10}$; $\frac{3}{4}$; $\frac{3}{7}$; $\frac{1}{2}$; $\frac{1}{4}$; $\frac{3}{11}$; $\frac{18}{23}$; $\frac{12}{17}$; $\frac{9}{13}$; $\frac{2}{3}$; $\frac{6}{11}$; $\frac{4}{7}$; $\frac{3}{7}$; $\frac{1}{2}$; $\frac{2}{5}$; $\frac{1}{3}$; $\frac{16}{19}$; $\frac{8}{11}$; $\frac{4}{9}$; $\frac{2}{7}$.

311–325. $\frac{20}{29}$; $\frac{10}{13}$; $\frac{5}{11}$; $\frac{4}{9}$; $\frac{2}{3}$; $\frac{36}{47}$; $\frac{24}{31}$; $\frac{18}{23}$; $\frac{12}{17}$; $\frac{9}{11}$; $\frac{8}{11}$; $\frac{6}{11}$; $\frac{4}{5}$; $\frac{3}{5}$; $\frac{2}{7}$.

CASE V.

¶ **75.** **326.** (p. 96.) The least common multiple of 2 & 4 is 4. (§ 62.) Hence $\frac{1}{\cancel{2}} \times \frac{\cancel{4}}{4} = \frac{2}{4}$. $\frac{1}{\cancel{4}} \times \frac{\cancel{4}}{4} = \frac{1}{4}$. $\frac{2}{4}$, $\frac{1}{4}$, Ans.*

327–345. $\frac{3}{6}$, $\frac{2}{6}$; $\frac{5}{10}$, $\frac{8}{10}$; $\frac{3}{6}$, $\frac{5}{6}$; $\frac{8}{12}$, $\frac{9}{12}$; $\frac{4}{6}$, $\frac{5}{6}$; $\frac{3}{9}$, $\frac{5}{9}$,

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* The rule given on p. 96, for the Reduction of Fractions to their least common denominator, and the operations according to that rule, are well adapted to furnish the pupil with a method of reduction which is easy, and which carries along with it an explanation of the process. After the pupil has become familiar with the principles of the rule, the following may be adopted, as being a little more concise: —

Find the least common multiple of the denominators. Divide this multiple by the denominator (of each fraction,) and multiply the quotient by the numerator. The product will be the numerator.

$\frac{9}{12}, \frac{2}{12}$; $\frac{15}{20}, \frac{14}{20}$; $\frac{12}{15}, \frac{7}{15}$; $\frac{16}{36}, \frac{21}{36}$; $\frac{10}{14}, \frac{2}{14}$; $\frac{1}{3}, \frac{1}{3}$; $\frac{3}{9}, \frac{5}{9}$; $\frac{16}{24}, \frac{21}{24}$; $\frac{15}{36}, \frac{26}{36}$; $\frac{15}{50}, \frac{36}{50}$; $\frac{3}{21}, \frac{5}{21}$; $\frac{10}{25}, \frac{7}{25}$; $\frac{16}{36}, \frac{15}{36}$.

346–360. $\frac{6}{12}, \frac{8}{12}, \frac{9}{12}$; $\frac{10}{30}, \frac{15}{30}, \frac{12}{30}$; $\frac{6}{12}, \frac{9}{12}, \frac{2}{12}$; $\frac{4}{8}, \frac{2}{8}, \frac{7}{8}$; $\frac{16}{24}, \frac{6}{24}, \frac{19}{24}$; $\frac{15}{35}, \frac{14}{35}, \frac{10}{35}$; $\frac{20}{36}, \frac{27}{36}, \frac{33}{36}$; $\frac{6}{36}, \frac{9}{36}, \frac{16}{36}$; $\frac{14}{36}, \frac{21}{36}, \frac{24}{36}$; $\frac{10}{36}, \frac{28}{36}, \frac{27}{36}$; $\frac{6}{45}, \frac{40}{45}, \frac{18}{45}$; $\frac{20}{28}, \frac{14}{28}, \frac{7}{28}, \frac{18}{28}$; $\frac{20}{35}, \frac{15}{35}, \frac{21}{35}$; $\frac{8}{12}, \frac{9}{12}, \frac{10}{12}$; $\frac{21}{24}, \frac{6}{24}, \frac{16}{24}$.

361–375. (p. 97.) $\frac{33}{44}, \frac{12}{44}$; $\frac{20}{36}, \frac{15}{36}$; $\frac{21}{24}, \frac{14}{24}$; $\frac{25}{30}, \frac{9}{30}, \frac{16}{30}$; $\frac{7}{42}, \frac{27}{42}, \frac{16}{42}$; $\frac{49}{70}, \frac{40}{70}, \frac{18}{70}$; $\frac{10}{20}, \frac{15}{20}, \frac{18}{20}$; $\frac{30}{60}, \frac{40}{60}, \frac{45}{60}, \frac{36}{60}, \frac{50}{60}$; $\frac{49}{105}, \frac{27}{105}$; $\frac{88}{231}, \frac{28}{231}$; $\frac{12}{90}, \frac{55}{90}, \frac{51}{90}$; $\frac{3}{6}, \frac{4}{6}, \frac{1}{6}$; $\frac{26}{91}, \frac{63}{91}$; $\frac{35}{105}, \frac{63}{105}, \frac{75}{105}$; $\frac{35}{40}, \frac{32}{40}, \frac{30}{40}$.

CASE VI. COMPLEX FRACTIONS.

The method of explaining the reduction of Complex Fractions, as given on pp. 97, 98, is here adopted, for the purpose of bringing all the operations in reduction together. An easier method, and one more intelligible to the pupil, is by the division of one fraction by another. After the pupil has become acquainted with the rule for the division of fractions, (p. 104,) it will be well for the teacher to recur to this section, and explain it by that rule.

¶ 76. **376.** (p. 98.) 1 reduced to a fractional form $=\frac{1}{1}$, & $2\frac{1}{2}$ reduced $=\frac{5}{2}$. Hence $\frac{1}{2\frac{1}{2}}$ is equivalent to $\frac{\frac{1}{1}}{\frac{5}{2}}$. Inverting the denominator $\frac{5}{2}$, &c., the expression becomes $\frac{1}{1} \times \frac{2}{5}, = \frac{2}{5}$, Ans.

377. $\frac{1}{2\frac{2}{3}} = \frac{\frac{1}{1}}{\frac{8}{3}} = \frac{1}{1} \times \frac{3}{8} = \frac{3}{8}$ Ans. Or by ¶ 81 $\frac{1}{1} \div \frac{8}{3} = \frac{3}{8}$ Ans., &c.

378. $\frac{2}{1\frac{7}{8}} = \frac{\frac{2}{1}}{\frac{15}{8}} = \frac{2}{1} \times \frac{8}{15} = \frac{16}{15} = 1\frac{1}{15}$, Ans.

379–395. $4\frac{4}{5}$; $\frac{3}{4}$; $\frac{1}{3}$; $\frac{1}{12}$; $1\frac{1}{2}$; $\frac{22}{27}$; $\frac{11}{15}$; $1\frac{1}{27}$; $\frac{10}{21}$; $\frac{14}{45}$; $19\frac{3}{5}$; $\frac{1}{33}$; $1\frac{5}{7}$; $2\frac{3}{4}$; $\frac{7}{8}$; $1\frac{8}{15}$; $1\frac{1}{14}$.

396–410. $\frac{5}{18}$; $\frac{5}{8}$; $\frac{5}{11}$; $\frac{6}{11}$; $\frac{1}{2}$; $\frac{28}{31}$; $1\frac{1}{3}$; 32; 49; 3; $\frac{1}{12}$; $\frac{1}{14}$; $\frac{16}{43}$; 4; $1\frac{1}{5}$.

411. $1\frac{1}{8}$.

412. $\frac{1}{2}$ of $\frac{3}{4}=\frac{3}{8}$. $\frac{2}{7}$ of $5=\frac{10}{7}$. Hence $\frac{\frac{1}{2}\text{ of }\frac{3}{4}}{\frac{2}{7}\text{ of }5}=\frac{\frac{3}{8}}{\frac{10}{7}}=\frac{3}{8}\times\frac{7}{10}=\frac{21}{80}$ Ans.

413–425. $\frac{32}{35}$; $\frac{27}{32}$; $\frac{29}{42}$; $\frac{24}{29}$; $1\frac{1}{7}$; $\frac{29}{31}$; $\frac{15}{17}$; $\frac{1}{2}$; $\frac{1}{8}$; 8; $\frac{15}{22}$; $\frac{2}{27}$; $11\frac{23}{24}$.

¶ **77.** **426.** (p. 99.) $\frac{5}{9}=\frac{?}{3}$. In this example, the fraction $\frac{5}{9}$ is to be changed to a fraction that shall have 3 for its denominator; and the question is, *If the denominator* 9, *of the given fraction, is to be changed to* 3, *to what must the numerator* 5 *be changed?* Now 3, the proposed denominator, is $\frac{3}{9}$ of 9, the given denominator; hence we are to take $\frac{3}{9}$ of the *given* denominator for the *proposed* denominator; and that the value of the fraction may not be changed, we must take $\frac{3}{9}$ also of the numerator; that is, we must multiply both terms of the given fraction by $\frac{3}{9}$, thus:— $\frac{5\times\frac{3}{9}}{\cancel{9}\times\frac{3}{\cancel{9}}}=\frac{\frac{15}{9}}{3}=\frac{1\frac{2}{3}}{3}$ Ans.*

427. $\frac{5}{9}=\frac{?}{2}$. In this example, 2 is $\frac{2}{9}$ of 9, &c. Hence multiply by $\frac{2}{9}$. $\frac{5\times\frac{2}{9}}{\cancel{9}\times\frac{2}{\cancel{9}}}=\frac{\frac{10}{9}}{2}=\frac{1\frac{1}{9}}{2}$ Ans.

426–435. See above. $\frac{1\frac{2}{3}}{3}$; $\frac{1\frac{1}{9}}{2}$; $\frac{2\frac{2}{9}}{4}$; $\frac{3\frac{8}{9}}{7}$; $\frac{6\frac{1}{9}}{11}$; $\frac{5\frac{5}{9}}{10}$; $\frac{3\frac{1}{3}}{6}$; $\frac{5}{9}$; $\frac{11\frac{1}{9}}{20}$; $\frac{6\frac{2}{3}}{12}$.

436. $\frac{7}{8}=\frac{5}{?}$. 5 is $\frac{5}{7}$ of 7; therefore multiply both terms of the fraction by $\frac{5}{7}$. $\frac{\cancel{7}\times\frac{5}{\cancel{7}}}{8\times\frac{5}{7}}=\frac{5}{\frac{40}{7}}=\frac{5}{5\frac{5}{7}}$ Ans.

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* This rule is not of very frequent practical use, but it involves an important principle in the reduction of fractions, and ought to be fully illustrated and explained.

437–445. $\frac{9}{10\frac{2}{7}}$; $\frac{2}{2\frac{2}{7}}$; $\frac{1}{1\frac{1}{7}}$; $\frac{4}{4\frac{4}{7}}$; $\frac{8}{9\frac{1}{7}}$; $\frac{11}{12\frac{4}{7}}$; $\frac{14}{16}$; $\frac{15}{17\frac{1}{7}}$; $\frac{19}{21\frac{5}{7}}$.

446–465. $\frac{9}{12}$; $\frac{5}{6\frac{2}{3}}$; $\frac{1}{1\frac{1}{3}}$; $\frac{8}{10\frac{2}{3}}$; $\frac{2}{2\frac{2}{3}}$; $\frac{16}{21\frac{1}{3}}$; $\frac{4}{5\frac{1}{3}}$; $\frac{6}{8}$; $\frac{20}{26\frac{2}{3}}$; $\frac{17}{22\frac{2}{3}}$; $\frac{7}{9\frac{1}{3}}$; $\frac{27}{36}$; $\frac{10}{13\frac{1}{3}}$; $\frac{11}{14\frac{2}{3}}$; $\frac{12}{16}$; $\frac{31}{41\frac{1}{3}}$; $\frac{42}{56}$; $\frac{13}{17\frac{1}{3}}$; $\frac{15}{20}$; $\frac{14}{18\frac{2}{3}}$.

466–485. $\frac{1\frac{2}{3}}{2}$; $\frac{2\frac{1}{2}}{3}$; $\frac{5\frac{5}{6}}{7}$; $\frac{10}{12}$; $\frac{3\frac{1}{3}}{4}$; $\frac{16\frac{2}{3}}{20}$; $\frac{9\frac{1}{6}}{11}$; $\frac{4\frac{1}{6}}{5}$; $\frac{6\frac{2}{3}}{8}$; $\frac{8\frac{1}{3}}{10}$; $\frac{10\frac{5}{6}}{13}$; $\frac{7\frac{1}{2}}{9}$; $\frac{20\frac{5}{6}}{25}$; $\frac{40\frac{5}{6}}{49}$; $\frac{11\frac{2}{3}}{14}$; $\frac{15}{18}$; $\frac{25}{30}$; $\frac{12\frac{1}{2}}{15}$; $\frac{15\frac{5}{6}}{19}$; $\frac{80}{96}$.

486–490. $\frac{\frac{8}{11}}{3}$; $\frac{1\frac{1}{17}}{9}$; $\frac{5}{23\frac{3}{4}}$; $\frac{3}{4\frac{10}{11}}$; $\frac{8}{92}$.

491–500. $\frac{4919}{164}$; $75\frac{22}{131}$; .19617 +; $\frac{13}{17}$; $\frac{57}{108}$, $\frac{14}{108}$, $\frac{48}{108}$; $\frac{4625}{1000000}$; $\frac{5605}{19}$; $\frac{\frac{32}{41}}{8}$; $\frac{1}{18}$; $\frac{28}{45}$.

¶ 78. 1–10. (p. 100.) $\frac{12}{3}$ or 4; $\frac{10}{6}$; $\frac{6}{4}$; $\frac{9}{5}$ or $1\frac{4}{5}$; $\frac{12}{7}$; $\frac{23}{7}$; $\frac{13}{8}$; $\frac{24}{11}$; $\frac{43}{23}$; $\frac{28}{6}$ or $4\frac{2}{3}$.

11–20. $2\frac{1}{12}$; $1\frac{1}{17}$; $2\frac{4}{13}$; $1\frac{7}{9}$; $6\frac{1}{2}$; $1\frac{14}{19}$; $\frac{11}{13}$; $5\frac{1}{8}$; $35\frac{2}{3}$; $18\frac{1}{2}$.

21–30. 10; $14\frac{1}{2}$; $11\frac{3}{4}$; $2\frac{4}{5}$; $10\frac{5}{6}$; $3\frac{6}{7}$; $8\frac{1}{3}$; $5\frac{7}{8}$; $9\frac{1}{9}$; 9.

31–40. $1\frac{1}{6}$; $2\frac{3}{4}$; $\frac{11}{12}$; $1\frac{2}{15}$; $1\frac{17}{60}$; $2\frac{13}{60}$; $1\frac{3}{14}$; $1\frac{7}{18}$; $1\frac{5}{18}$; $1\frac{3}{28}$.

41–50. $14\frac{1}{4}$; $4\frac{3}{4}$; $7\frac{1}{2}$; 5; $46\frac{11}{12}$; $20\frac{1}{3}$; $1\frac{17}{21}$; $7\frac{5}{36}$; $15\frac{19}{36}$; $12\frac{11}{18}$.

51–60. $10\frac{2}{3}$; $12\frac{13}{24}$; $7\frac{7}{24}$; $6\frac{34}{77}$; $2\frac{20}{63}$; $2\frac{13}{60}$; $7\frac{7}{22}$; $15\frac{5}{8}$; $17\frac{5}{18}$; $3\frac{18}{35}$.

¶ 79. 61–70. (p. 102.) $\frac{3}{7}$; $\frac{7}{19}$; $\frac{4}{13}$; $\frac{5}{10}$; $4\frac{1}{2}$; $7\frac{2}{3}$; $8\frac{2}{3}$; $6\frac{3}{4}$; $7\frac{1}{2}$; 19.

71–80. $5\frac{1}{4}$; $6\frac{3}{4}$; $2\frac{3}{5}$; $4\frac{3}{5}$; $3\frac{4}{5}$; $4\frac{3}{5}$; $\frac{3}{4}$; $1\frac{1}{4}$; $\frac{5}{8}$; $7\frac{7}{8}$.

81–90. $1\frac{1}{6}$; $7\frac{1}{4}$; $8\frac{1}{3}$; $4\frac{1}{12}$; 6; $7\frac{5}{6}$; $8\frac{3}{4}$; $9\frac{11}{12}$; $5\frac{2}{3}$; $4\frac{7}{12}$.

91–110. $1\frac{5}{18}$; $4\frac{2}{9}$; $6\frac{1}{6}$; $3\frac{1}{9}$; $3\frac{1}{18}$; $\frac{17}{18}$; $5\frac{8}{9}$; $6\frac{5}{6}$; $8\frac{7}{9}$; $4\frac{13}{18}$; $7\frac{2}{3}$; $5\frac{11}{18}$; $3\frac{5}{9}$; $4\frac{1}{2}$; $8\frac{4}{9}$; $\frac{7}{18}$; $5\frac{5}{12}$; $7\frac{3}{4}$; $4\frac{4}{5}$; $2\frac{13}{15}$.

111–120. $3\frac{6}{7}$; $\frac{11}{14}$; $4\frac{5}{14}$; $8\frac{3}{8}$; $\frac{13}{30}$; $5\frac{7}{18}$; $\frac{5}{12}$; $7\frac{5}{12}$; $1\frac{17}{20}$; $1\frac{4}{5}$.

¶ 80. 121–130. (p. 103.) $\frac{1}{8}$; $\frac{1}{12}$; $2\frac{6}{7}$; $\frac{12}{7}$; $\frac{18}{7}$; 1; $\frac{5}{8}$; $\frac{63}{80}$; $\frac{4}{7}$; $\frac{35}{64}$.

131–140. $\frac{3}{28}$; 1; $2\frac{1}{3}$; $8\frac{17}{18}$; $26\frac{2}{3}$; $5\frac{5}{8}$; $10\frac{1}{2}$; $1\frac{1}{6}$; $31\frac{1}{6}$; $3\frac{2}{3}$.

141–150. 1; $\frac{2}{3}$; 1; 2; 2; 6; $1\frac{8}{21}$; $15\frac{3}{5}$; $\frac{49}{64}$; $4\frac{5}{8}$.

151–160. $12\frac{5}{6}$; $9\frac{1}{3}$; 35; 7; 43; 23; 23; 11; 57; $\frac{7}{22}$; $2\frac{1}{2}$.

161–170. 55; 45; 129; 1; 35; 29; 37; 23; 57; 2.

171–180. 3; 1; 7; 1; 5; 1; $16\frac{1}{3}$; $38\frac{17}{28}$; $3\frac{23}{24}$; $\frac{20}{57}$.

DIVISION OF FRACTIONS.

The rule for the Division of Fractions may be illustrated as follows:—

Divide $\frac{3}{5}$ by $\frac{2}{7}$. In the first place, divide $\frac{3}{5}$ by the whole number 2. As we cannot divide the numerator, without a remainder, we multiply the denominator, thus: $\frac{3}{5\times2}=\frac{3}{10}$. Now, $\frac{2}{7}$ is only $\frac{1}{7}$ as large as 2, and, if used as a divisor, will give a quotient 7 times larger than that obtained above. We therefore multiply $\frac{3}{10}$ by 7, thus: $\frac{3\times7}{10}=\frac{21}{10}$. By examining the steps of the above process, it will be seen that we have multiplied the numerator of the dividend by the denominator of the divisor, and the denominator of the dividend by the numerator of the divisor; or, in other words, we have *inverted the divisor*, and multiplied its terms into those of the dividend.

¶ 81. 181–190. (p. 104.) $1\frac{1}{4}=\frac{5}{4}$. $\frac{5}{4}\div\frac{1}{4}=\frac{5}{4}\times\frac{4}{1}=\frac{20}{4}$, or 5. 5; 7; 8; 9; 11; 36; 14; 2; 22; 31.

191–200. 23; $7\frac{2}{3}$; $3\frac{2}{7}$; $2\frac{1}{11}$; $1\frac{5}{18}$; $1\frac{1}{22}$; $1\frac{11}{12}$; $\frac{115}{152}$; $\frac{7}{8}$; $\frac{23}{40}$.

201–210. $\frac{13}{18}$; $\frac{11}{16}$; $\frac{4}{33}$; $\frac{21}{34}$; $\frac{34}{35}$; $\frac{39}{43}$; $\frac{10}{19}$; $\frac{9}{16}$; $\frac{47}{54}$; $\frac{27}{38}$.

211–220. $\frac{1}{4}$; $\frac{13}{21}$; $\frac{1}{3}$; 4; $2\frac{4}{33}$; $\frac{5}{7}$; $\frac{17}{32}$; $\frac{2}{21}$; 9; $\frac{32}{61}$.

221–230. 5; $\frac{410}{623}$; $\frac{24}{55}$; $\frac{19}{56}$; 2; $\frac{1}{3}$; $\frac{1}{3}$; $\frac{105}{248}$; $\frac{27}{50}$; $\frac{9}{11}$.

231–240. $\frac{5}{168}$; $\frac{169}{288}$; $1\frac{17}{63}$; 208; $\frac{2}{5}$; $\frac{74}{143}$; $\frac{259}{754}$; $1\frac{9}{20}$; $\frac{11}{560}$; $\frac{1253}{8136}$.

INFINITE DECIMALS.

¶ **82.** **241.** (p. 105.) $1 \div 7 = .\dot{1}4285\dot{7}$. Proof, $\frac{142857}{999999} = \frac{1}{7}$.

242–250. $.\dot{2}8571\dot{4}$; $.\dot{4}2857\dot{1}$; $.\dot{5}7142\dot{8}$; $.\dot{7}1428\dot{5}$; $.\dot{8}5714\dot{2}$; $.08\dot{3}$; $.41\dot{6}$; $.58\dot{3}$; $.91\dot{6}$.

251–270. $.1\dot{6}$; $.8\dot{3}$; $.\dot{0}\dot{9}$; $.\dot{2}\dot{7}$; $.\dot{6}\dot{3}$; $.\dot{8}\dot{1}$; $.\dot{9}\dot{0}$; $.0\dot{5}$; $.2\dot{7}$; $.3\dot{8}$; $.6\dot{1}$; $7\dot{2}$; $.9\dot{4}$; $.041\dot{6}$; $.208\dot{3}$; $.291\dot{6}$; $.458\dot{3}$; $.541\dot{6}$; $.708\dot{3}$; $.791\dot{6}$.

271. *$.1\dot{5} = \frac{1\frac{5}{9}}{10} = \frac{14}{90} = \frac{7}{45}$ Ans.

272. $.2\dot{3}\dot{0} = \frac{2\frac{30}{99}}{10} = \frac{228}{990} = \frac{38}{165}$ Ans.

273–280. $5\frac{1}{6}$; $\frac{1}{300}$; $1\frac{3}{110}$; $\frac{41}{220}$; $\frac{2707}{9990}$; $4\frac{34}{45}$; $4\frac{25}{33}$; $\frac{5}{6}$.

281–290. $7\frac{2}{3}$; $41\frac{1}{3}$; $\frac{83}{100}$; 16; $4\frac{1}{11}$; $5\frac{2}{55}$; $\frac{8}{495}$; $7\frac{83}{99}$; $\frac{388}{4950}$; $\frac{907}{9900}$.

291. $.\dot{1}\dot{6} = \frac{16}{99}$. Then, $18 \times \frac{16}{99} = \frac{288}{99} = 2\frac{10}{11}$ Ans.

292. $28.\dot{5} = 28\frac{5}{9} = \frac{257}{9}$. $13.\dot{1}\dot{6} = 13\frac{16}{99} = \frac{1303}{99}$. Then, $\frac{257}{9} : \frac{1303}{99} = \frac{2827}{1303} = 2\frac{221}{1303}$ Ans.

293–300. $3\frac{88}{105}$; 3; $7\frac{242}{405}$; $\frac{2501}{4456}$; $\frac{17}{8100}$; $35\frac{1}{12}$; $1\frac{13}{27}$; $\frac{9}{770}$.

MISCELLANEOUS EXERCISES IN FRACTIONS.

¶ **83.** **301–310.** (p. 106.) 43; 3; 15; 11; 13; 19; 21; 29; 22; 80.

311–320. $\frac{112}{4}$; $\frac{196}{7}$; $\frac{308}{11}$; $\frac{420}{15}$; $\frac{616}{22}$; $\frac{1092}{39}$; $\frac{1764}{63}$; $\frac{2240}{80}$; $\frac{2800}{100}$; $\frac{7000}{250}$.

321–330. $\frac{5}{18}$; $\frac{7}{9}$; 12; $5\frac{8}{11}$; $3\frac{1}{4}$; 1; $2\frac{3}{4}$; $29\frac{5}{7}$; $1\frac{1}{8}$; $33\frac{3}{4}$.

331–340. **(¶ 76.)** $\frac{1}{2}$; $\frac{19}{43}$; $\frac{2}{19}$; $\frac{7}{19}$; $\frac{1}{21}$; $4\frac{1}{2}$; $1\frac{13}{20}$; $\frac{47}{64}$; $\frac{5}{14}$; $13\frac{1}{2}$.

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* $.1\dot{5}$ is equivalent to 1 tenth and $\frac{5}{9}$ of another tenth; hence we write it as a complex fraction, thus: $\frac{1\frac{5}{9}}{10}$.

341–350. $6\frac{1}{8}$; $4\frac{1}{2}$; $10\frac{4}{9}$; $10\frac{7}{8}$; $15\frac{5}{12}$; $8\frac{3}{4}$; $7\frac{7}{9}$; $7\frac{11}{36}$; $1\frac{31}{36}$; $18\frac{1}{15}$.

351–360. $\frac{27}{55}$; $\frac{6}{7}$; $\frac{35}{44}$; $\frac{1}{20}$; $\frac{33}{196}$; $1\frac{29}{63}$; $2\frac{17}{24}$; $\frac{19}{85}$; $9\frac{1}{3}$; $6\frac{14}{27}$.

361–370. $5\frac{7}{11}$; $7\frac{4}{13}$; $7\frac{7}{19}$; $10\frac{7}{8}$; $14\frac{14}{15}$; $9\frac{1}{17}$; $42\frac{14}{23}$; $34\frac{13}{29}$; $55\frac{1}{2}$; $32\frac{9}{31}$.

371–380. $\frac{39}{5}$; $\frac{94}{7}$; $\frac{113}{9}$; $\frac{92}{11}$; $\frac{71}{13}$; $\frac{113}{6}$; $\frac{149}{10}$; $\frac{403}{17}$; $\frac{934}{19}$; $\frac{6051}{56}$.

381–390. $\frac{2}{3}$; $\frac{9}{26}$; $\frac{3}{4}$: $\frac{7}{11}$; $\frac{7}{19}$; $\frac{87}{401}$; $\frac{339}{524}$; $\frac{5}{13}$; $\frac{109}{181}$; $\frac{41}{247}$.

391. $\frac{1}{2}=\frac{5}{?}$. 5 is $\frac{5}{1}$ of 1. Hence multiply both terms of $\frac{1}{2}$ by $\frac{5}{1}$ or 5. Thus, $\frac{1\times 5}{2\times 5}=\frac{5}{10}$ Ans. See ¶ **77.**

392–400. $\frac{2}{4\frac{2}{3}}$; $\frac{13}{23\frac{5}{6}}$; $\frac{1}{\frac{2}{15}}$; $\frac{4}{\frac{3}{4}}$; $\frac{19}{120\frac{1}{3}}$; $\frac{7}{40\frac{1}{4}}$; $\frac{2}{\frac{16}{59}}$; $\frac{1}{\frac{11}{61}}$; $\frac{9}{21}$.

401–410. $1\frac{1}{18}$; $6\frac{4}{9}$; $9\frac{13}{36}$; $\frac{7}{30}$; $4\frac{25}{28}$; $5\frac{7}{10}$; $1\frac{16}{45}$; $2\frac{9}{14}$; $3\frac{19}{60}$; $43\frac{38}{105}$.

411–420. (p. 107.) (¶ **82.**) $\frac{2}{15}$; $2\frac{7}{90}$; $\frac{28}{33}$; $7\frac{62}{99}$; $13\frac{21}{22}$; $\frac{397}{495}$; $\frac{331}{900}$; $1\frac{1}{2}$; $\frac{31}{100}$; $\frac{241}{275}$.

421–430. $.8\dot{3}$; $.2105+$; $.2890625$; $.6923+$; $.4\dot{6}$; $.9\dot{6}$; $.80208\dot{3}$; $.0875$; $.1$; $.\dot{0}6\dot{3}$.

431–440. 9.84, 98.4, 984, 9840; 6.84; 192.3, 1923; 4820; 7.5; 7500.

441–450. $\frac{42}{105}$, $\frac{50}{105}$; $\frac{32}{616}$, $\frac{91}{616}$; $\frac{70}{231}$, $\frac{55}{231}$; $\frac{147}{42}$, $\frac{35}{42}$, $\frac{33}{42}$, $\frac{16}{42}$; $\frac{45}{99}$, $\frac{88}{99}$; $\frac{52}{65}$, $\frac{40}{65}$; $\frac{21}{70}$, $\frac{5}{70}$, $\frac{38}{70}$; $\frac{153}{187}$, $\frac{99}{187}$; $\frac{65}{117}$, $\frac{45}{117}$; $\frac{60}{105}$, $\frac{70}{105}$, $\frac{42}{105}$.

451–460. (¶ **77.**) $\frac{2\frac{1}{2}}{5}$; $\frac{2\frac{4}{7}}{9}$; $\frac{\frac{6}{17}}{2}$; $\frac{2\frac{2}{3}}{3}$; $\frac{8\frac{4}{5}}{11}$; $\frac{24}{9}$; $\frac{\frac{4}{17}}{4}$; $\frac{4\frac{1}{8}}{11}$; $\frac{3\frac{6}{7}}{9}$; $\frac{129\frac{6}{11}}{15}$.

461–470. $22\frac{2}{3}$; $11\frac{2}{3}$; $8\frac{4}{11}$; 32; 59; 1143; 56.4; $38\frac{7}{54}$; 23; $2\frac{7}{13}$.

471–480. $3\frac{1}{3}$ bu.; $7\frac{5}{6}$ bu.; $2\frac{14}{15}$ bu.; 5 bu.; 8 bu.; $4.75; 12\frac{2}{3}$; $16.625; $52.25; $27.90625.

481–485. $5.50; $4.80; 5\frac{1}{3}$; $2.75; $4.

486–490. (p. 108.) $\frac{8}{19}$ h.; $\frac{4}{5}$ h.; $\frac{10}{19}$ h.; $\frac{16}{25}$ h.; $\frac{44}{73}$ h.

491–500. $13\frac{1}{3}$ mi.; $11\frac{1}{3}$ mi.; 20 mi.; $10\frac{1}{3}$ mi.; 34 mi.; $3\frac{1}{4}$ mi.; 10 mi.; $13\frac{3}{4}$ mi.; $21\frac{2}{3}$ mi.; $19\frac{1}{2}$ mi.

501–510. $\$4\frac{5}{16}$; \$34.50; \$69; $\$32\frac{11}{32}$; \$40.25; $\$0.18\frac{3}{4}$; \$0.25; $\$0.37\frac{1}{2}$; $\$0.31\frac{1}{4}$; \$0.20.

511–520. 4 bu.; $2\frac{2}{15}$ bu.; $\frac{1}{3}$ bu.; $\frac{8}{15}$ bu.; $17\frac{7}{9}$ bu.; 6 cts.; $4\frac{1}{2}$ cts.; $5\frac{1}{4}$ cts.; $6\frac{3}{4}$ cts.; $10\frac{1}{2}$ cts.

521–530. $2\frac{2}{3}$ r.; 20 r.; 10 r.; $17\frac{13}{27}$ r.; $21\frac{1}{3}$ r.; 1 r.; $1\frac{7}{9}$ r.; 14 r.; $35\frac{5}{9}$ r.; $7\frac{11}{27}$ r.

531–540. \$36; \$234; \$171; \$310.50; \$552; \$345; \$172.80; \$409.50; \$204; \$235.50.

VIII. COMPOUND NUMBERS.

¶ 98. 1–10. (p. 114.) 36; 44; 64; 108; 76; 176; 140; 200; 300; 156.

11–20. 24; 36; 48; 60; 84; 108; 168; 180; 238; 588.

21–30. 80; 40; 140; 180; 260; 300; 400; 500; 740; 980.

31–40. 2; 3; 6; 9; 12; 15; 24; 30; 64; 148.

41–50. 2; 3; 6; 9; 15; 20; 23; 32; 41; 56.

51–60. 2; 4; 6; 9; 11; 18; 29; 32; 36; 49.

61–70. 103; 113; 47; 17; 106, 424; 80, 960, 3840; 1488.

71–80. 12 d. 1 qr. = 1 s. 1 qr.; 41 s., or £2 1 s.; 40 s. = £2; 36 d. = 3 s.; 250 d. = 20 s. 10 d.

81–90. 2; 3; 5; 8; 11; $12\frac{1}{2}$; $15\frac{5}{8}$; $24\frac{7}{12}$; $30\frac{11}{12}$; 40.

91–100. (p. 115.) 3; 5; 7; $10\frac{1}{2}$; 22; 43; $47\frac{1}{2}$; 37; 45; $31\frac{1}{2}$.

101–110. 2; 4; 6; 8; 10; 16; $20\frac{5}{6}$; $25\frac{5}{6}$; $43\frac{1}{4}$; $53\frac{11}{12}$.

111–120. 60; 71; 120; 129; 183; 91; 106; 140; 76; 162.

121–130. 80; 140; 153; 199; 131; 118; 584; 951; 1497; 1036.

131–140. 72; 89; 119; 139; 262; 367; 332; 141; 282; 1977.

141–150. 40; 47; 59; 78; 108; 153; 221; 290; 268; 518.

151–160. 40; 100; 99; 153; 237; 294; 225; 458; 716; 2852.

161–170. 300 ℈; 100 ʒ; 12$\frac{1}{2}$ ℥; 1$\frac{1}{24}$ ℔; 233$\frac{1}{3}$ ʒ; 29$\frac{1}{6}$ ℥; 2$\frac{31}{72}$ ℔; 62$\frac{1}{2}$ ℥; 5$\frac{5}{24}$ ℔; 39$\frac{1}{6}$ ℔.

171–180. 80; 97; 103; 235; 89; 173; 319; 256; 210; 158.

181–190. 84; 196; 215; 335; 323; 636; 820; 501; 1469; 443.

191–200. 128; 256; 304; 377; 586; 1280; 1568; 1020; 1911; 2304.

201–210. 5 oz. 10 dr.; 12 oz. 8 dr.; 1 lb. 14 oz.; 2 lb. 4oz.; 3 lb. 3 oz. 14 dr.; 2 lb. 13 oz. 9 dr.; 2 lb. 2 oz. 9 dr.; 3 lb. 6 oz. 8 dr.; 3 lb. 10 oz.; 3 lb. 8 oz. 2 dr.

211–220. (p. 116.) 3 qr. 13 lb.; 2 qr. 16 lb.; 3 qr.; 1 cwt. 3 qr.; 2 cwt. 1 qr. 18 lb.; 2 cwt. 3 qr.; 5 cwt.; 6 cwt. 3 qr.; 2 T. 3 cwt. 3 qr. 5 lb.; 3 T. 8 cwt. 16 lb.

221–230. 6; 4; 12; 16; 16$\frac{2}{3}$; 45; 77$\frac{11}{12}$; 83$\frac{1}{4}$; 360; 730$\frac{1}{4}$.

231–240. 6; 13; 16; 29; 33; 66$\frac{2}{3}$; 83$\frac{1}{3}$; 146$\frac{2}{3}$; 185; 293.

241–250. 2; 3; 4; 7; 18; 24; 20$\frac{4}{5}$; 21$\frac{7}{8}$; 24$\frac{3}{40}$; 24$\frac{19}{40}$.

251–260. 3; 9; 12; 22; 18$\frac{3}{4}$; 29$\frac{3}{8}$; 59$\frac{3}{4}$; 115$\frac{3}{8}$; 68$\frac{3}{4}$; 48$\frac{3}{8}$.

261–270. 40 fur. = 1600 r.; 62 fur. = 2480 r.; 151 fur. = 6040 r.; 1429 r.; 14 ft. = 168 in.; 14220 in.

271–280. 3200; 4480; 1920; 1419; 5105; 2560; 4340; 8320; 25640; 50560.

281–290. 121; 242; 605; 2389$\frac{3}{4}$; 2571$\frac{1}{4}$; 9680; 1936; 2904; 6050; 9680.

291–300. 3; 6; 5; 6$\frac{1}{2}$; 10$\frac{5}{12}$; 12; 18$\frac{23}{24}$; 24; 68$\frac{7}{12}$; 60$\frac{25}{36}$.

301–310. 320 r., 8 R., 2 A.; 420 r., 10$\frac{1}{2}$ R., 2$\frac{5}{8}$ A.; 360 r., ., 2$\frac{1}{4}$ A.; 423 r., 10$\frac{23}{40}$ R., 2$\frac{103}{160}$ A.; 550 r., 13$\frac{3}{4}$ R., 3$\frac{7}{16}$ A.;

1600 r., 40 R., 10 A.; 2610 r., $65\frac{1}{4}$ R., $16\frac{5}{16}$ A.; 1332 r., 33.3 R., $8\frac{13}{40}$ A.; 1320 r., 33 R., $8\frac{1}{4}$ A.; 490 r., $12\frac{1}{4}$ R., $3\frac{1}{16}$ A.

311–320. 81; 135; 189; 351; 370; 188; 560; 233; 133; 345.

321–330. 3456; 8640; 13824; 5324; 32832; 8632; 51840; 138240; 150278; 186624.

331–340. (p. 117.) $1\frac{21}{64}$ ft., $\frac{85}{1728}$ yd.; $3\frac{403}{864}$ ft., $\frac{2995}{23328}$ yd.; $3\frac{1}{8}$ ft., $\frac{25}{216}$ yd.; $31\frac{1}{4}$ ft., $1\frac{17}{108}$ yd.; $44\frac{307}{432}$ ft., $1\frac{7651}{11664}$ yd.; $55\frac{155}{864}$ ft., $2\frac{1019}{23328}$ yd.; $51\frac{1637}{1728}$ ft., $1\frac{43109}{46656}$ yd.; $53\frac{845}{864}$ ft., $1\frac{23309}{23328}$ yd.; $72\frac{73}{216}$ ft., $2\frac{3961}{5832}$ yd.; $51\frac{1647}{1728}$ ft., $1\frac{43119}{46656}$ yd.

341–350. 12; 28; 44; 52; 63; 69; 118; 350; 373; 195.

351–360. 40; 96; 31; 22; 152; 80; 61; 172; 282; 343.

361–370. 14; 30; 74; 139; 86; 126; 256; 112; 160; 219.

371–380. 1 pk. 1 qt. 1 pt.; 1 bu.; 3 bu.; 6 bu.; 5 bu. 3 pk. 1 qt.; 2 bu. 3 pk. 2 qt. 1 pt.; 8 bu. 7 qt. 1 pt.; 12 bu. 3 pk. 7 qt. 1 pt.; 19 bu. 2 pk. 7 qt. 1 pt.; 46 bu. 1 pk. 6 qt.

381–390. 20 qt., 40 pt.; 48 qt., 96 pt.; 68 qt., 136 pt.; 111 qt., 222 pt.; 38 qt., 76 pt.; 125 qt., 250 pt.; 176 qt., 352 pt.; 504 qt., 1008 pt.; 756 qt., 1512 pt.; 194 qt., 388 pt.

391–400. 126; 315; 441; 945; 1260; 533; 312; 992; 3780; 6111.

401–410. 28; 108; 252; 96; 608; 1504; 4032; 184; 244; 190.

411–420. 16 pt., 8 qt., 2 gall.; 40 pt., 20 qt., 5 gall.; $69\frac{1}{2}$ pt., $34\frac{3}{4}$ qt., $8\frac{11}{16}$ gall.; 88 pt., 44 qt., 11 gall.; 128 pt., 64 qt., 16 gall.; 104 pt., 52 qt., 13 gall.; 216 pt., 108 qt., 27 gall.; 225 pt., $112\frac{1}{2}$ qt., $28\frac{1}{8}$ gall.; $134\frac{1}{4}$ pt., $67\frac{1}{8}$ qt., $16\frac{25}{32}$ gall.; $329\frac{3}{4}$ pt., $164\frac{7}{8}$ qt., $41\frac{7}{32}$ gall.

421–430. 64 qt., 16 gall.; 370 qt., $92\frac{1}{2}$ gall., $1\frac{59}{126}$ hhd.; 417 qt., $104\frac{1}{4}$ gall., $1\frac{55}{84}$ hhd.; 558 qt., $139\frac{1}{2}$ gall., $2\frac{3}{14}$ hhd.; 690 qt., $172\frac{1}{2}$ gall., $2\frac{31}{42}$ hhd.; $798\frac{1}{2}$ qt., $199\frac{5}{8}$ gall., $3\frac{85}{504}$ hhd.; 2160 qt., 540 gall., $8\frac{4}{7}$ hhd.; 750 qt., $187\frac{1}{2}$ gall., $2\frac{41}{42}$ hhd.; 1260 qt., 315 gall., 5 hhd.; 2268 qt., 567 gall., 9 hhd.

431–440. 3; 12; 15; $17\frac{4}{15}$; $19\frac{1}{15}$; $31\frac{1}{5}$; 60; 73; 150; $120\frac{59}{60}$.

441–450. 60; 210; 330; 148; 287; 249; 313; 179; 222; 352.

451–460. (p. 118.) 1080; 3240; 4320; 6480; 9720; 5040; 3960; 6120; 1260; 2789.

461–470. 8 min., $\frac{2}{15}$ h.; 20 min., $\frac{1}{3}$ h.; 42 min., $\frac{7}{10}$ h.; 49 min., $\frac{49}{60}$ h.; $66\frac{2}{3}$ min., $1\frac{1}{9}$ h.; $158\frac{1}{3}$ min., $2\frac{23}{36}$ h.; $163\frac{13}{15}$ min., $2\frac{329}{450}$ h.; $76\frac{37}{60}$ min., $1\frac{997}{3600}$ h.; 180 min., 3 h.; 663 min., $11\frac{1}{20}$ h.

471–480. 3; 5; 32; 65; 2; 5; 60; 365; $246\frac{2}{3}$; $553\frac{1}{4}$.

481–490. 1825; 4380; 2555; 6935; 9125; 4745; 1820; 3200; 1200; 4800.

491–500. 2820; 2280; 10800; 61200; 172800; 1036800; 460800; 28740; 119820; 126144000.

REDUCTION OF COMPOUND NUMBERS.

¶ 100. **501–510.** (p. 120.) 93; 147; $103\frac{1}{2}$; 285; 488; 1368; 2.28; 432; 1715; 3780.

511–520. 1 oz. 12 dwt. 1 gr.; 8 lb. 11 oz.; 37 lb. 5 oz.; 3 lb. 3 oz. 3 dwt.; 6 lb. 5 oz.; 1 lb. 8 oz. 9 dwt. 11 gr.; 8 lb. 3 oz. 3 dwt.; 35 lb. 9 dwt.; 41 lb. 7 oz. 19 dwt.; 16 lb. 8 oz. 3 dwt. 11 gr.

521–530. 10 ℥ 4 ʒ 2 ℈ $2\frac{6}{17}$ gr.; 7 ʒ 12 gr.; 18 cwt. 3 qr. 8 lb. 3 oz. $12\frac{4}{17}$ dr.; 3 qr. 20 lb. 2 oz. 8.96 dr.; 24 lb. 5 oz. 12.16 dr.; 2 fur. 4 r. 1 yd. $5\frac{13}{18}$ in.; 23 r. 2 ft. $7\frac{5}{19}$ in.; 3 qr. $1\frac{5}{7}$ na.; 355 A. 2 R. 8 r. 26 yd. 8 ft.; 13 r. 22 yd. 8.91 ft.

531–540. $\frac{217}{360}$; $\frac{29}{60}$; $\frac{253}{360}$; $\frac{67}{72}$; $\frac{19}{24}$; $\frac{3}{40}$; $\frac{1}{576}$; $\frac{7}{270}$; $\frac{31}{36}$; $\frac{23}{60}$.

541–550. (p. 121.) .046875; .21875; .828125; .4375; .625; .171875; .265625; .84375; .515625; .703125.

551–560. 88; 164; 27; 6048; 1890; 261; 12064; 8088; 1764; 4038.

561–570. 2 yd. 2 ft. 832 in.; 20 yd. 3 ft.; 5 cords, 5 cord ft. 9 cu. ft.; 55 tons 25 ft.; 235 tons 30 ft.; 2° 25′ 20″; 156° 40′; 13 circ. 116°; 1 dy. 3 h. 28 min. 50 sec.; 5 y. 46 dy. 16 h.

571–580. 10 h. 54 min. $32\frac{8}{11}$ sec.; 34 min. $17\frac{1}{7}$ sec.; 55′ 48″; 2 pk. 6.4 qt.; 5 gall. 2 qt. 1 pt. 1.44 gi.; 35 cu. ft.; 1 R. 5 r. 21 yd. 5 ft. $66\frac{6}{7}$ in.; 23 yd. 2 ft. $60\frac{12}{13}$ in.; 1 fur. 16 r. 2 yd. $1\frac{13}{17}$ ft.; 13 s. 4 d.

581–590. .3; .075; .15; .525; .6; .95; $.291\dot{6}$; $.\dot{6}$; .8; $.40\dot{5}$.

591–600. $\frac{127}{320}$; $\frac{1}{384}$; $\frac{7}{3200}$; $\frac{157}{1760}$; $\frac{95}{63360}$; $\frac{151}{63360}$; $\frac{111}{440}$; $\frac{323}{2640}$; $\frac{1}{352}$; $\frac{313}{320}$.

ADDITION OF COMPOUND NUMBERS.

¶ 102. 602–610. (p. 122.) £9 15 s.; £7 12 s. 4 d.; £7 15 s. 11 d.; £10 18 s. 9 d.; 9 lb. 8 oz. 2 dwt.; 11 lb. 11 oz. 16 dwt. 17 gr.; 1 lb. 3 oz. 14 dwt. 12 gr.; 6 ℔ 11 ℥ 3 ʒ 1 ℈; 1 ℔ 6 ℥ 5 ʒ 1 ℈ 16 gr.

611–620. 3 T. 5 cwt. 2 qr. 6 oz.; 1 T. 2 qr. 13 lb. 11 oz.; 1 qr. 1 lb. 11 oz. 13 dr.; 1 T. 1 qr. 1 lb.; 2 m. 4 fur. 16 r.; 1 m. 39 r. 4 yd.; 7 yd. 1 ft. 4 in.; 22 yd. 1 qr. 2 na.; 1 yd. 3 qr. 2 na.; 8 yd. 2 qr. 1 na.

621–625. 6 m. 72 A. 13 r.; 60 yd. 4 ft. 73 in.; 7 m. 86 A. 1 R.; 20 yd. 14 ft.; 32 cords.

626–630. 17 bu. 2 pk. 2 qt.; 1 bu. 2 pk. 4 qt.; 5 pk. 5 qt.; 1 circ. 41° 18′ 26″; 3 circ. 152° 35″.

631–635. 6 hhd. 58 gall. 1 qt. 1 pt.; 1 hhd. 1 pt. 1 gi.; 11 gall. 2 gi.; 1 hhd. 45 gall. 3 qt.; 1 hhd. 59 gall. 1 qt.

636–640. 9 y. 242 dy. 10 h.; 1 dy. 12 h. 17 sec.; 13 y. 257 dy.; 1 dy. 18 h. 4 min.; 7 y. 26 dy. 9 h.

SUBTRACTION OF COMPOUND NUMBERS.

¶ 104. 642–650. (p. 123.) £2 6 s. 3 d.; 3 d. 2 qr.; 7 s. 4 d. 2 qr.; £1 19 s. 8 d.; £2 19 s. 11 d. 2 qr.; 1 lb.

8 dwt.; 3 lb. 11 oz. 19 dwt. 6 gr.; 1 lb. 5 oz.; 3 oz. 8 dwt. 6 gr.

651–660. 9 ℥; 1 ℔ 10 ℥ 1 ʒ; 1 ℥ 2 ʒ 2 ℈ 5 gr.; 1 qr.; 1 cwt. 1 qr. 1 lb.; 3 qr. 18 lb. 8 oz. 1 dr.; 19 cwt. 3 qr. 23 lb. 2 oz. 11 dr.; 1 T. 16 cwt. 2 qr.; 1 m. 8 r.; 7 fur. 39 r. 4 yd. 2 ft. 6 in.

661–670. 5 yd. 1 in.; 2 m. 2 fur. 29 r.; 1 yd. 3 qr. 3 na.; 2 yd. 3 qr. 1 na.; 5 yd. 2 qr. 2 na.; 4 sq. m. 639 A. 3 R. 35 r.; 636 A. 3 R.; 23 yd. 6 ft. 41 in.; 26 ft. 88 in.; 1 bu. 5 qt. 1 pt.

671–680. 3 pk. 1 pt.; 1 pt.; 1 qt.; 8 gall. 2 qt. 1 pt.; 300° 31′ 33″; 82 y. 8 mo. 3 dy.; 4 yr. 10 mo. 13 dy.; 6 y. 9 mo. 28 dy.; 111 y. 2 mo. 11 dy.; 282 y. 6 mo. 8 dy.

COMPOUND MULTIPLICATION.

¶ 106. 682–690. £26 16 s. 3 d.; £32 16 s. 3 d.; £11 12 s. 9 d.; £67; £40 9 s. 7 d.; 23 lb. 5 oz. 2 dwt.; 1 lb. 11 oz. 19 dwt.; 22 lb. 3 oz. 2 dwt. 9 gr.; 174 lb. 8 oz. 2 dwt.

691–700. 18 ℔ 8 ℥ 3 ʒ; 4 ℔ 6 ℥ 5 ʒ 1 ℈; 2 ℥ 4 ʒ 2 ℈ 10 gr.; 151 ℔ 3 ℥ 4 ʒ 1 ℈ 10 gr.; 44 ℔ 5 ℥ 2 ʒ; 35 T. 12 cwt. 2 qr.; 5 cwt. 12 lb. 4 oz.; 2 qr. 15 lb. 14 oz. 14 dr.; 14 m. 4 fur. 10 r.; 97 m. 2 fur.

701–710. 2 fur. 35 r. 3 yd. 1 ft. 2 in.; 344 yd. 1 qr.; 78 yd. 3 qr.; 241 yd. 3 na.; 63 A. 18 r.; 40 bu. 1 pk. 5 qt.; 8 bu. 2 pk. 2 qt. 1 pt.; 24 bu. 3 pk. 7 qt. 1 pt.; 29 bu. 3 pk. 1 pt.; 93 bu. 1 pk. 4 qt.

711–720. 24 hhd. 51 gall. 3 qt.; 22 gall. 1 qt. 1 gi.; 8 hhd. 35 gall. 2 gi.; 239 hhd. 60 gall.; 2 circ. 130° 10′ 39″; 41 y. 49 dy. 17 h.; 116 dy. 5 h. 4 min.; 1 dy. 23 h. 58 min. 21 sec.; 19 y. 7 mo.; 145 y. 27 dy.

COMPOUND DIVISION.

¶ 108. 722–730. (p. 127.) £5 7 s. 3 d.; £4 13 s. 9 d.; 19 s. 4d. 3 qr.; £8 7 s. 6 d.; £4 11 d. 2 qr.; 3 lb. 10 oz. dwt.; 11 oz. 19 dwt. 12 gr.; 7 lb. 5 oz. 19 gr.; 19 lb.
z. 18 dwt.

731–735. 3 ℔ 8 ℥ 7 ʒ; 6 ℥ 6 ʒ 2 ℈; 2 ℈ 10 gr.; 5 ℔ 4 ʒ 9 gr.; 4 ℔ 11 ℥ 2 ʒ.

736–740. 5 T. 18 cwt. 3 qr.; 2 qr. 25 lb. 12 oz.; 2 lb. 10 oz. 10 dr.; 7 fur. 30 r.; 2 m. 3 fur. 18 r.

741–750. 28 r. 4 yd. 2 ft. 11 in.; 9 yd. 2 qr. 1 na.; 7 yd. 3 qr. 2 na.; 14 yd. 3 na.; 5 A. 2 R. 38 r.; 13 bu. 1 pk. 7 qt.; 3 pk. 6 qt. 1 pt.; 2 pk. 4 qt. 1 pt.; 7 qt. 1 pt.; 5 bu. 6 qt.

751–760. 4 hhd. 60 gall. 3 qt.; 2 qt. 1 pt. 3 gi.; 7 gall. 1 pt. 2 gi.; 19 hhd. 62 gall. 3 qt.; 94° 27′ 51″; 5 yr. 319 dy. 23 h.; 14 dy. 12 h. 38 min.; 5 h. 19 min. 49 sec.; 7 mo. 25 dy.; 2 y. 3 mo. 19 dy.

761–770. 5; 31; 75; 12; 9; 7; 8; 9; 30; 63.

MISCELLANEOUS EXAMPLES.

¶ 109. 771–780. (p. 127.) 127 yd. 1 qr. 1 na.; £250 4 s. $6\frac{1}{4}$ d.; 12 cwt. 3 qr. 21 lb. 11 oz.; 92 bu. 6 qt. 1 pt.; 12 m. 3 fur. 38 r. 2 yd. 2 ft.; 12 T. 11 cwt. 1 qr. 14 lb. 13 oz.; 86 bu. 2 pk. 1 qt. 1 pt.; 176 m. 6 fur. 39 r.; 152 lb. 5 oz. 7 dwt. 18 gr.; 3 yd. 2 qr. $1\frac{1}{5}$ na.

781–794. 2 oz. 16 dwt. $18\frac{1}{6}$ gr.; 23 m. 26 r. 3 yd. 2 ft.; 1 bu. $3\frac{8}{13}$ qt.; $4\frac{156}{1543}$; 4 lb. 1 oz. 3 dwt. 23 gr.; 516 gall. 1 qt. 1 pt.; 12 yd. 1 na.; £49 16 s. 11 d. 1 qr.; 2 cwt. 11 lb. $5\frac{5}{47}$ oz.; 4179 gall. 2 gi.; $1180\frac{17}{32}$; £115 4 s. $8\frac{1}{4}$ d.; 5 ℔ 9 ℥ 1 ʒ 9 gr.; 1 ℔ 1 ℥ 6 ʒ 2 ℈ 13 gr.

795. 31 × 30 = 930 r. = (÷ 40 & by 4,) 5 A. 3 R. 10 r. sold.

A.	R.	r.
149	3	0
5	3	10
143 A.	3 R.	30 r. left.

796. 48 × 8 × 5 = 1920 cu. ft. 1920 ÷ 128 = 15 cords, original pile. 8 × 5 × 6 = 240 cu. ft. = (÷ 16 & by 8,) 1 cd. 7 cd. ft. sold.

15	
1	7
13 cds.	1 cd. ft. left.

797–798. 107 gall. 2 qt. 1 pt.; 12 s. 8 d. $2\frac{2}{31}$ qr.

799. 11 × 4 × 6 × 17 = 4488 cu. ft. = (÷ 128,) 35 cds. 8 cu. ft. Ans.

800–801. 1 A. 1 R. $2\frac{14}{33}$ r.; 334 dy.

802. $48\frac{1}{2} \times 2 \times 1\frac{1}{2} = 145\frac{1}{2}$ cu. ft. in each log. $145\frac{1}{2} \times 13 = 1891\frac{1}{2}$ cu. ft. $1891\frac{1}{2} \div 40 = 47$ T. $11\frac{1}{2}$ ft. Ans.

803–815. 44 m. 1 r.; 14 y. 8 mo. 1 dy.; 28 y. 1 mo. 13 dy.; 3 y. 5 mo. 15 dy.; 789.12; 1 T. $6\frac{5}{19}$ cu. ft.; 1 cwt. 3 qr. = 196 lb.; $\frac{7}{9}$; $\frac{47471}{86400}$; $\frac{3}{8}$; $\frac{15}{16}$; $\frac{7}{15}$.

816–830. $.46458\dot{3}$; .375; $.\dot{1}3\dot{6}$; $\frac{7}{8}$; .84375; 1 qr. 21 lb.; 9 s. 7.44 d.; 11 mo. $2\frac{44}{53}$ dy.; 17 gall. $1\frac{5}{11}$ pt.; 5 h. 20 min.; \$0.375; 5 fur. 3 r. 3 yd. 1 ft. 6 in.; 10 oz. $5\frac{5}{7}$ dwt.; 3 qr. 3 lb. 11.136 oz.; 5 mo. 15 dy.

831. $142.5 \times 119\frac{3}{4} = 17064\frac{3}{8}$ sq. rods. $17064\frac{3}{8} \div 160 = 106\frac{167}{256}$ A. in the pond. $106\frac{167}{256}$ A. — $13\frac{2}{7}$ A. = $93\frac{657}{1792}$ A. Ans.

832. 763.4 × 763.4 = 582779.56 sq. ft. 1 A. = 43560 sq. ft. (¶ **92.**) 582779.56 ÷ 43560 = $13\frac{3409}{9000}$ Acres.

833. $\frac{5}{8}$ A. = 2 R. 20 r. $\frac{2}{7}$ A. = 1 R. $5\frac{5}{7}$ r.

A.	R.	r.
49	3	31
56	2	20
119	0	27
38	1	$5\frac{5}{7}$
264 A.	0 R.	$3\frac{5}{7}$ r.

$3\frac{5}{7}$ r. ÷ 160 = $\frac{13}{560}$ A.
Then, $264\frac{13}{560}$ A. × \$28.50 = \$7524.66 Ans.

834. 2997 m. 3 fur. 28 r. = 15826602 ft. $\frac{1}{54000} \times 15826602 = \frac{15826602}{54000}$ drs. = $293.085\dot{2}$ drs. = 1 lb. 2 oz. $5.085\dot{2}$ drs. Ans.

835–838. £514 10 s. 7 d.; 456 bu.; 96264000; $65\frac{1133}{1472}$ cts.

839. (p. 131.) $8.3 \times 3.5 \times 4\frac{1}{4} \times 19 = 2345\frac{63}{80}$ cu. ft. = (÷ 128) 18 cords $41\frac{63}{80}$ cu. ft., = ($41\frac{63}{80}$ cu. ft. = $\frac{3343}{10240}$ cds.) $18\frac{3343}{10240}$ cds. Then, $\$5.375 \times 18\frac{3343}{10240}$ = \$98.50 Ans.

840. $19\frac{4}{7}$ cwt. = 2192 lb. 2192 × $.06\frac{1}{2}$ = \$142.48. \$156.75 — \$142.48 = \$14.27 Ans.

841. 3 qr. 2 na. = $\frac{7}{8}$ yd. = .875 yd. 95.875 × \$3.50 = \$335.56$\frac{1}{4}$. \$335.56 + \$35.75 = \$371.31$\frac{1}{4}$ Ans.

842–845. 5460; 37; 1 s. $1\frac{17}{43}$ d.; 22 A. 2 R. $13\frac{2}{3}$ r.

846. $\frac{4}{5}$ of $\frac{7}{8}$ of $32\frac{1}{3} = \frac{679}{30}$. $\frac{1}{11}$ of $\frac{6}{7}$ of $49 = \frac{42}{11}$. $\frac{679}{30} - \frac{42}{11} = \frac{6209}{330}$ yds. = 18 yd. 3 qr. $1\frac{7}{165}$ na. Ans.

847–848. \$$1\frac{5}{63}$; \$1.33$\frac{1}{3}$.

849. The fractions reduced are, $\frac{99}{5}$, $\frac{23}{8}$, $\frac{10}{9}$, $\frac{119}{120}$, $\frac{956}{15}$, and their least common denominator is 360, &c. The fractions added = $88\frac{23}{45}$ Ans.

850. 49 T. 5 cwt. 1 qr. = 110348 lb. 110348 ÷ 196 = 563 bbls. \$4.875 × 563 = \$2744.625. Ans.

IX. PERCENTAGE.

¶ 111. 1–10. (p. 132.) 5.22; 3.84; 18.96; 55.92; 6.162; 23.6436; 594; 10.9542; .00354; .3144.

11–20. 9; 33.75; .1431; .00405; 6.3; 2.9115; 442.35; 540; .3915; 6.885.

21–30. .2475; .7425; 2.2; 8.25; 19.25; .0026125; 26.125; 55; .91$\frac{2}{3}$; .34375.

31–40. .24; .36; .0052; .18125; .0096875; 1.25; 2.1875; .134375; $\frac{1}{8}$; .09$\frac{3}{8}$.

41–50. \$156.25; \$208.33$\frac{1}{3}$; \$312.50; \$416.66$\frac{2}{3}$; \$625; \$833.33$\frac{1}{3}$; \$937.50; \$1666.66$\frac{2}{3}$; \$2187.50; \$2500.

51–60. $8\frac{1}{3}$; $12\frac{1}{2}$; $16\frac{2}{3}$; 25; $33\frac{1}{3}$; $37\frac{1}{2}$; $66\frac{2}{3}$; $87\frac{1}{2}$; 100.

61–70. 46.3925; \$13.75; \$9.35; \$0.031625; 22.3575; .0026125; 129.8; \$0.462; 247.5; 1.98.

71–90. 50; $62\frac{1}{2}$; $83\frac{1}{3}$; $41\frac{2}{3}$; $6\frac{1}{4}$; $18\frac{3}{4}$; $31\frac{1}{4}$; $43\frac{3}{4}$; $56\frac{1}{4}$; $68\frac{3}{4}$; $81\frac{1}{4}$; $93\frac{3}{4}$; 28; $\frac{8}{10}$; 20; $\frac{4}{25}$; 92; $77\frac{7}{8}$; $42\frac{6}{7}$; $88\frac{4}{17}$.

91–100. 10.88; 47; 12.5; 85; 11.57; 2.1875; 15; .1075; 1.224; 40.48.

COMMISSION.

¶ 112. **101.** (p. 133.) \$1963.50 × $.04\frac{1}{2}$ = \$88.36 commission. \$1963.50 — \$88.36 = \$1875.14 paid over.

102. \$9550 × $3\frac{1}{3}$ per ct. = \$318.33 commission. \$9550 — \$318.33 = \$9231.67 Ans.

103. $2\frac{3}{4}$ per ct. of \$4762 = \$130.955 Ans.

104. Performed.

105. (p. 134.) £153 7 s. 11 d.

$$\begin{array}{rrr}
 & & .025 \\
\hline
\pounds 3.825 & .175 & .275 \\
20 & & \\
\cline{1-1}
16500 & & \\
175 & & \\
\cline{1-1}
16.675\text{ s.} & & \\
12 & & \\
\cline{1-1}
8100 & & \\
275 & & \\
\cline{1-1}
8.375\text{ d.} & &
\end{array}$$

8.375 d. $.375 = \frac{3}{8}$. £3 16 s. $8\frac{3}{8}$ d. Ans.

INSURANCE.

¶ 113. **106.** (p. 134.)

£144 8 s. 6 d.

$$\begin{array}{rrr}
 & & .02 \\
\hline
\pounds 2.88 & .16 & .12 \\
20 & & \\
\cline{1-1}
1760 & & \\
16 & & \\
\cline{1-1}
17.76\text{ s.} & & \\
12 & & \\
\cline{1-1}
912 & & \\
12 & & \\
\cline{1-1}
9.24\text{ d.} & &
\end{array}$$

9.24 d. $.24 = \frac{6}{25}$. £2 17 s. $9\frac{6}{25}$ d. Ans.

107–108. \$215.49; \$20.25.

109. $1\frac{3}{4} = \frac{7}{4}$. £5163 10s.

```
                .07
   ---------------
   4) 361.41  .70
   ---------------
   £90–1.41
          20        £90   7 s.   2.7 d.    Premium.
    --------                3    6          Policy.
        2820        ------------------
          70        £90  10 s.   8.7 d.    Ans.
     -------
     4) 28.90
     -------
       7–.90 s.
          12
    --------
    4) 10.80
    --------
       2.7 d.
```

110. \$10875 + 4500 = \$15375. \$4650 + 1750 = 6400.
$\frac{3}{4}$ per cent. of \$15375 = \$115.31$\frac{1}{4}$.
$\frac{1}{2}$ " " \$6400 = \$32.00.
$\frac{5}{8}$ " " \$975 = \$6.09$\frac{3}{8}$.
\$153.41 Ans.

TAXES.

¶ 114. **111–113.** \$71; \$125.835; \$40.375.

114. .5 + .1 + $\frac{1}{5}$ (= .2) = .8 per cent. .8 per cent. of \$7500 = \$60 Ans.

115. $\frac{7}{8}$ per cent. of \$2775 = \$24.28 Ans.

STOCKS.

¶ 115. **116.** (p. 135.) \$100 × 27 = \$2700. \$1.00 + .165 = \$1.165. \$2700 × 1.165 = \$3145.50 Ans.

117. \$100 × 31 = \$3100. \$1.00 — .09$\frac{1}{4}$ = .90$\frac{3}{4}$. \$3100 × .90$\frac{3}{4}$ = \$2813.25 Ans.; or thus: 9$\frac{1}{4}$ per cent. of \$3100 = \$286.75. \$3100 — \$286.75 = \$2813.25 Ans.

118–121. \$4025; \$11569.69; \$11875, \$7663$\frac{1}{3}$, \$4750; \$121.50.

DUTIES.

[Allowances for draft and leakage are supposed to have been made, except in Ex. 126.]

¶ 116. **122.** (p. 136.) 5 cwt. 3 qr. 11 lb. = 655 lb. 12 per cent. of 655 = 78.6. 655 — 78.6 = 576.4. 576.4 lb. × .03 = $17.29 Ans.

123–127. $35.25 ; $5.62 ; $137.50 ; $157.61 ; $52.42.

LOSS AND GAIN.

¶ 117. **128.** .90 — .75 = .15. .15 ÷ .75 = 20 per cent. Ans.

129–130. $11.25, or 20 per cent. ; 15 per cent.

131. 40 per cent of $6.50 = $2.60. $6.50 + 2.60 = $9.10. $9.10 ÷ 112 = .08$\frac{1}{8}$ Ans.

132. 2240 — 2000 = 240. 240 ÷ 2240 = 10$\frac{5}{7}$ per cent.

133. 112 — 100 = 12. 12 ÷ 112 = 10$\frac{5}{7}$ per cent.

134. 13 cwt. 3 qr. 17 lb. = 1557 lb. 1557 × .09$\frac{1}{2}$ = $147.915. $147.915 — $125 = $22.915 loss. $22.915 ÷ 147.915 = 15$\frac{1}{2}$ per cent. Ans.

135–140. 25 ; 25 ; 20$\frac{5}{6}$; 25 ; 34$\frac{2}{3}$; 14$\frac{2}{7}$.

INTEREST.

¶ 118. **141–150.** (p. 138.) $18 ; $27 ; $.165 ; $5.88 ; $45.72 ; $7.17 ; $5.241 ; $2.94 ; $6.90 ; $37.80.

151–160. $20 ; $35 ; $100 ; $0.059 ; $0.034 ; $0.215 ; $0.925 ; $17.25 ; $2.67 ; $9.35.

161–170. $10 ; $30 ; $0.70 ; $50 ; $0.33 ; $23.28 ; $3.4952 ; $70.40 ; $0.9872 ; $6.072.

171–180. $9.10 ; $3.1675 ; $1.022 ; $6.37 ; $56 ; $0.0679 ; $31.50 ; $11.3981 ; $3.325 ; $2.3709.

181–190. $32.83 ; $65.66 ; $229.79 ; $147.72 ; $218.85 ; $111.25 ; $14.92 ; $20.52 ; $192.86 ; $8.66.

191–200. $53.66 ; $120.73 ; $404.66 ; $40.24 ; $50.68 ; $304.05 ; $203.89 ; $218.20 ; $460.55 ; $347.27.

¶ **119.** **201.** (p. 139.) 3 yr. × .06 = .18 interest of $1 for 3 years. $100 × .18 = $18 Ans.

202. 5 yr. 6 mo. $= 5\frac{1}{2}$ or $\frac{11}{2}$ yr. $.06 \times \frac{11}{2} = .33$. $100 × .33 = $33 Ans.

203. 11 mo. 27 dy. = 357 dy. $= \frac{357}{360}$ yr. $\$100 \times .06 \times \frac{357}{360}$* = $5.95 Ans.

204. 7 yr. 3 mo. $= 7\frac{1}{4}$ or $\frac{29}{4}$ yr. $\frac{\$100 \times .06 \times 29}{1 \times 1 \times 4} = \$43.50$ Ans.

205. 1 mo. 15 dy. $= \frac{1\frac{1}{2}}{12}$ yr. $= \frac{1}{8}$ yr. $\frac{1}{8} \times \frac{.06}{1} \times \$100 =$ $0.75 Ans.

206. 2 yr. 4 mo. 15 dy. $= 2\frac{135}{360}$, or $2\frac{3}{8}$ yr. $= \frac{19}{8}$ yr. $\frac{\$100 \times 19 \times .06}{8}$ = $14.25 Ans.

207. 7 mo. 10 dy. $\frac{220}{360}$ yr. $= \frac{11}{18}$ yr. $\frac{\$100 \times 11 \times .06}{18} =$ $\$3.66\frac{2}{3}$ Ans.

208. $100 × 13 × .06 = $78 Ans.

209. $\frac{7}{12} \times \frac{.06}{1} \times \frac{\$100}{1} = \$3.50$ Ans.

210. 5 yr. 5 dy. $= 5\frac{1}{72}$ yr., or $\frac{361}{72}$ yr. $\$100 \times .06 \times \frac{361}{72} = \$30.08$ Ans.

211–220. $28.50 ; $53.04 ; $36.10 ; $15.83 ; $95 ; $308.75 ; $468.35 ; $151.21 ; $47.50 ; $516.80.

221–230. $4 ; $9 ; $27 ; $\$54\frac{1}{3}$; $410 ; $210 ; $80 ; $75 ; $743 ; $590.

231–240. (p. 140.) $3.75 ; $2.34 ; $9.375 ; $79.69 ; $318.75 ; $375 ; $156.61 ; $17.19 ; $62.34 ; $150.

241–250. $252 ; $15 ; $450 ; $29.40 ; $130.50 ; $105 ; $31 ; $14.70 ; $225 ; $192.30.

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* In examples under this rule it will be advisable to express all the operations and then perform them at once. It will readily be perceived that this rule affords frequent opportunities for cancelling, especially in operations at 6 per cent. Thus, to obtain the multiplier for 7 mo. $\frac{7}{\underset{2}{\cancel{12}}} \times \frac{\cancel{6}}{1} = \frac{7}{2}$ or $3\frac{1}{2}$ cts.; and for 2 mo. 12 dy. (= 72 dy.) $= \frac{\overset{6}{\cancel{72}}}{\underset{5}{\underset{\cancel{60}}{\cancel{360}}}} \times \frac{\cancel{6}}{1} = \frac{6}{5}$, &c.

BANK AND BUSINESS RULE.

¶ 120. **251–260.** (p. 140.) 500 × 33 = 16500. 16500 ÷ 6000 = $2.75 Ans.; $\frac{500\times 63}{6000} = \$5.25$ Ans.; $7.75; $10.25; $7.50; $12.50; $17.50; $27.50; $16.50; $12.25.

261–270. (p. 141.) $13.30; $16.625; $47.25; $75.25; $27.30; $11.90; $19.95; $94.50; $3.675; $16.22.

271–280. $\frac{\$100\times 94}{6000} = \$1.566$ interest at 6 per cent. $\frac{7}{6}$ of 1.566 = $1.83 Ans.; $1.24; $2.41; $3.50; $1.75; $11.08; $23.92; $15.38; $2.625; $4.63.

281–290. $1.31; $0.89; $1.72; $2.50; $1.25; $7.92; $17.08; $10.99; $1.875; $3.31.

291–300. $3.67; $7; $10.33; $13.67; $10; $16.67; $23.33; $36.67; $22; $16.33.

¶ 121. PARTIAL PAYMENTS.

Legal Rule.

Compute interest upon the principal from the date of the note (or the time when it commences bearing interest) to the time of the first payment. If this interest is less than the payment then made, add it to the principal, and from the amount deduct the payment. The remainder will form a new principal, upon which interest is to be cast to the time of the next payment. This interest must be added to the last principal, and from it the payment deducted as before. So continue to the time of settlement.

But if in any case the interest is greater than the payment, deduct the payment from it and reserve the excess. Then compute interest upon the same principal, unaltered, to the time of the next payment, and add such interest to the excess before reserved. So continue until the payment exceeds the interest due, when proceed as above.

¶ 122. 301. (p. 143.)

Principal on interest from July 4th, 1844, . .		$500.00
Interest from July 4, 1844, to Sept. 5, 1844, (2 mo. 1 dy. = 61 dy.) . .		5.08
Amount Sept. 5th, 1844, ...		$505.08
Deduct first payment, ...		97.25
Balance for new principal, ...		$407.83
Interest from Sept. 5, 1844, to June 3, 1845, (8 mo. 28 dy. = 268 dy.) .	18.22	
Deduct second payment, less than interest due, ...	15.75	
Excess of interest to be reserved, . .	$2.47	
Interest from June 3, 1845, to Aug. 1, 1845, (1 mo. 28 dy. = 58 dy.) . .	3.94	6.41
Amount due Aug. 1, 1845, ...		414.24
Deduct third payment, ...		217.00
Balance for new principal, ...		197.24
Interest from Aug. 1, 1845, to Jan. 1, 1846, (5 mo.)		4.93
Amount due Jan. 1, 1846, ...		$202.17 Ans.
302. Principal on interest from May 5, 1846, ...		$862.50
Interest from May 5, to Sept. 29, 1846, (4 mo. 24 dy. = 144 dy.)	$20.70	
First payment, less than interest due, .	17.	
Excess of interest to be reserved, . .	3.70	
Interest from Sept. 29, 1846, to April 17, 1847, (6 mo. 18 dy.)	28.46	
Interest due April 17, 1847, ...	32.16	
Second payment less than interest due, .	27.50	
Excess of interest, &c., ...	4.66	
Interest from April 17, to Dec. 13, 1847, (7 mo. 26 dy. = 236 dy.) . .	33.93	38.59
Amount due Dec. 13, 1847, ...		$901.09
Deduct third payment, ...		800.

Balance for new principal, (Brot. over.) .	$101.09	
Interest from Dec. 13, 1847, to Sept. 3, 1848, (8 mo. 20 dy. = 260 dy.) .	4.38	
Amount due Sept. 3, 1848, ...	$105.47	Ans.

303. Principal on int. from Jan. 1, 1847,	$1000.00	
Int. (at 7 per cent.) from Jan. 1, to Sept. 25, 1847, (8 mo. 24 dy. = 264 dy.) .	51.33	
Amount due Sept. 25, 1847, ...	1051.33	
Deduct first payment, ...	243.50	
Balance for new principal, ...	807.83	
Interest from Sept. 25, to Dec. 13, 1847, (2 mo. 18 dy. = 78 dy.)	12.25	
Amount, &c., ...	820.08	
Deduct second payment, ...	15.89	
Balance for new principal, ...	804.19	
Interest from Dec. 13, 1847, to May 5, 1848, (4 mo. 22 dy. = 142 dy.) . .	22.20	
Amount, ...	826.39	
Deduct third payment, ...	34.75	
Balance for new principal, ...	791.64	
Interest from May 5, 1848, to Oct. 1, 1848, (4 mo. 26 dy. = 146 dy.)	22.47	
Amount due Oct. 1, 1848.	$814.11	Ans.

COMPOUND INTEREST.

¶ 124. 304. (p. 144.) Rate for 6 mo. = .03. Rate for 4 mo. 22 dy. $= \frac{142}{360}$ yr. $\times \frac{6}{1} =$ .0236.

Principal	$200	
200 × .03 =	6	Int. for 6 mo.
	206	Amt. for 6 mo.
206 × .03 =	6.18	Int. for 6 mo.
	212.18	Amt. for 1 year.
212.18 × .03 =	6.37	Int. for 6 mo.

(Brot. up.)	218.55	Amt. for 1 yr. 6 mo.
218.55 × .0236 =	5.15	Int. for 4 mo. 22 dy.
	$223.70*	Ans. for 1 yr. 10 mo. 22 dy.

305–310. $167.78; $419.44; $559.26; $41.66; $104.58; $720.33.

311. (p. 145.) Rate for 6 mo., .025.

Principal	$180.	
180 × .025 =	4.50	Int. for 6 mo.
	184.50	Amt. for 6 mo.
184.50 × .025 =	4.61	Int. for 6 mo.
	$189.11	Amt. for 1 yr. Ans.

312–315. $203.65; $211.88; $196.26; $189.04.

316. Principal $500.

500 × .05 =	25.	Int. for 1 year.
	525.	Amt. for 1 year.
525 × .05 =	26.25	Int. for 1 year.
	551.25	Amt. for 2 years.
551.25 × .05 =	27.56	Int. for 1 year.
	578.81	Amt. for 3 years.
578.81 × .05 =	28.94	Int. for 1 year.
	607.75	Amt. for 4 years.
Deduct principal	500.	
	$107.75	Comp. Int. Ans.*

317–320. $85.56; $209.26; $110.85; $69.18.

PROBLEMS IN INTEREST.

¶ 125. **321.** (p. 145.) Interest on $500, 1 yr. 6 mo. at 1 per cent. = $7.50. $37.50 ÷ 7.50 = 5 Ans.

322–330. 4½; 6; 5; 4; 4½; 7; 8; 5⅓; 6¼.

331. Interest on $1500 for 1 yr. = $105. $420 ÷ $105 = 4 yr. Ans.

332. $1044 — $900 = $144 int. Interest on $900 for 1 yr = $54. 144 ÷ 54 = 2⅔ yr., or 2 yr. 8 mo. Ans.

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* See Appendix, "Compound Interest."

333. $510 — $500 = $10 int. Interest on $500 for 1 yr. = $25. $10 ÷ $25 = $\frac{2}{5}$ yr., or 4 mo. 24 dy. Ans.

334–340. 11 mo. 10 dy.; $16\frac{2}{3}$; 25; 20; $14\frac{2}{7}$; $18\frac{2}{11}$; $14\frac{22}{27}$.

341. Interest of $1 for 18 mo. = .09. $150 ÷ .09 = $\$1666\frac{2}{3}$ Ans.

342–350. $27500; $19875; $26250; $\$8727\frac{3}{11}$; $\$7272\frac{8}{11}$; $\$13090\frac{10}{11}$; $\$29090\frac{10}{11}$; $\$5133\frac{128}{187}$; $\$7480\frac{40}{77}$.

351. (p. 147.) Amount of $1 for 1 yr. 3 mo. = $1.075. $537.50 ÷ 1.075 = $500 Ans.

352. Amount of $1 for 2 yr. 4 mo. 15 dy. at 7 per cent. = 1.16625. $1399.50 ÷ 1.16625 = $1200 Ans.

353–360. $2475; $4923; $1834.86; $1785.71; $1673.64; $1913.88; $1888.57; $1837.95.

DISCOUNT.

¶ 126. 361. Amount of $1 for 1 yr. and 9 mo. = 1.105. 900 ÷ 1.105 = $814.48 present worth. $900 — $814.48 = 85.52 discount. Ans.

362–363. $114.49; $343.75.

364. Amount of $1 for 9 mo. 27 dy. = 1.05775. 700 ÷ 1.05775 = 661.78. 700 — 661.78 = $38.22 Ans.

365–370. $160.71; $64.59; $123.85; $61.84; $244.77; $111.11.

BANK DISCOUNT.*

¶ 127. 371. $\frac{\$2000 \times 33}{6000}$ = $11. discount. $2000 — $11 = $1989 Ans.

372–376. $492.25; $372.94; $185.53; $734.625; $455.58.

377–391. (p. 149.) $\frac{500 \times 93}{6000}$ = 7.75, bank discount at 6 per cent. $\frac{5}{6}$ of 7.75 = 6.46 discount at 5 per cent. $500 — $6.46 = $493.54 Ans. $373.28; $185.86; $737.19;

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* See Appendix, "Bank Discount."

$456.77; $490.96; $372.59; $185.20; $732.06; $454.38; $489.67; $372.25; $184.87; $729.50; $453.19.

392–394. $8705.89; $1190.025; $962.72.

395. Interest on $2000 for 123 dy. at $5\frac{1}{4}$ per cent. $35.88. 2000 — 35.88 = 1964.12. $\frac{1}{7}$ of $\frac{3}{8} = \frac{3}{56}$. $\frac{3}{56}$ of $1964.12 = $105.22 Ans.

396. Rate for 93 dy. at 4 per cent. $= \frac{93}{360} \times .\frac{04}{1} = .\frac{31}{30}$. (£63 17 s. 6 d.) × $.\frac{31}{30}$ = (p. 133.) 13 s. $2\frac{1}{2}$ d. £63 17 s. 6 d. — 13 s. $2\frac{1}{2}$ d. = £63 4 s. $3\frac{1}{2}$ d. Ans.

397. $\frac{487.50 \times 63}{6000}$ = $5.12 Bank discount. Amount of $1. for 63 dy. 1.0105. 487.5 ÷ 1.0105 = 482.43, Present worth. $487.50 — $482.43 = $5.07, Equitable discount. $5.12 — $5.07 = .05, Bank discount. Ans.

398. Interest, or Bank discount, of $2000 at 5 per cent. for 4 mo. (123 dy.) = $34.17. $2000 — $34.17 = $1965.83, Present worth. $1965.83 — $1600 = $365.83 Ans.

399. $163\frac{5}{8} \times .87\frac{1}{2} =$ $143.17
$127\frac{1}{2} \times .62\frac{1}{2} =$ 79.69
$97\frac{3}{4} \times .93\frac{3}{4} =$ 91.64
$225 \times .50 =$ 112.50
$115\frac{5}{8} \times .37\frac{1}{2} =$ 43.36
$470.36 Amt. of note.

Interest on $470.36 for 63 dy. $4.94. $470.36 — $4.94 = $465.42 Ans.

400. $6\frac{4}{7} \times 112 \times .05\frac{1}{2} =$ $40.48
$81 \times 4.625 =$ 374.63
$3.375 \times 112 \times .08\frac{5}{8} =$ 32.60
2 cwt. 1 qr. 17 lb. = $2\frac{45}{112}$ cwt., × 3.50 = 8.41
$13 \times 32\frac{1}{2} \times .29\frac{3}{4} =$ 125.69
$581.81 Amt. of note.

Interest of $581.81 at 7 per cent. for 93 days = $10.52. And $581.81 — $10.52 = $571.29 Ans.

X. ANALYSIS AND RATIO.

¶ 129. **1–10.** (p. 153.) $\frac{18 \times 15}{9} = 30$ w.; 27 w.; 45 w.; 54 w.; 9 w.; 10 w.; 6 w.; 5 w.; 3 w.; 2 w.

11–20. $\frac{36 \times 35}{6} = 210$; 180; 42; 90; 84; 60; 126; 45; 30; 20.

21–30. $\frac{15 \times 112}{70} = 24$; 9; 30; 6; 18; 27; 12; 21; 180; 42.

31–40. $\frac{\$85.50 \times 13}{19} = \$58.50$; \$121.50; \$378; \$238.50; \$223.87½; \$30.94; \$445.50; \$168.75; $\$15\frac{3}{16}$; \$4.22.

41–50. $\frac{26 \times 9}{13} = 18$ dy.; 39 dy.; 13 dy.; 9 dy.; 6 dy.; $6\frac{1}{2}$ dy.; $9\frac{3}{4}$ dy.; $4\frac{1}{3}$ dy.; 3 dy.; 2 dy.

51–60. $\frac{3 \times 4}{3} = 4$; 6; 2; 2.4; $1\frac{1}{2}$; $2\frac{10}{19}$. (57) $\frac{3 \times 4^*}{\frac{15}{16}} =$ 12.8; $3\frac{3}{31}$; $2\frac{2}{11}$; $2\frac{2}{3}$.

61–70. $\frac{9.5 \times 6}{5.75}$ or $\frac{19 \times 6 \times 4}{2 \times \quad 23} = 9\frac{21}{23}$; $8\frac{1}{7}$; $7\frac{1}{8}$; $14\frac{1}{4}$; $8\frac{10}{13}$; $7\frac{5}{21}$; $17\frac{7}{13}$; $12\frac{3}{14}$; $6\frac{1}{3}$; $7\frac{3}{5}$.

71–80. $\frac{28.125 \times 3}{4.5} = \$18.75$; \$43.75; \$34.375; \$39.06; \$73.44; \$118.75; \$593.75; \$111.875; $\$51.56\frac{1}{4}$; \$5.86.

81–90. (p. 154.) $\frac{9.5 \times 8}{5} = 15\frac{1}{5}$; 19; 38; $12\frac{2}{3}$; $6\frac{1}{3}$; $8\frac{4}{9}$; $25\frac{1}{3}$; $10\frac{6}{7}$; 7.6; 6.

91–100. $\frac{12 \times 12}{9\frac{3}{5} \text{ or } 9.6} = 15$ ft.; $22\frac{1}{2}$ ft.; $11\frac{1}{4}$ ft.; 3 ft. 9 in.; $8\frac{3}{4}$ ft.; $11\frac{1}{8}$ ft.; $6\frac{3}{32}$ ft.; $13\frac{3}{4}$ ft.; $33\frac{3}{4}$ ft.; $20\frac{5}{6}$ ft.

¶ 131. **101–110.** (p. 155.) 7 : 9; 15 : 3; 8 : 11, &c.

111–120. $\frac{3}{5}$; $\frac{11}{2}$; $\frac{1}{8}$, &c.

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* In examples of this kind containing a fraction for a denominator, which makes the statement a complex fraction, the fractional divisor may be *inverted* (**¶ 81**) and placed on a line with the other terms, thus: $\frac{3 \times 4}{\frac{15}{16}} = \frac{3 \times 4 \times 16}{15} = 12.8$.

121–130. .5; .75; 1.3 +; .875; 1.6; 2.6 +; .125; .9375; 1.1875; .40625.

131–140. $\frac{1}{2}$; $\frac{1}{5}$; $\frac{3}{4}$; $\frac{3}{5}$; 3; 9; $\frac{5}{3}$; $\frac{3}{4}$; $\frac{11}{6}$; $\frac{5}{9}$.

141–150. £5 = 100 s. 3 : 100 or $\frac{3}{100}$; $\frac{3}{8}$; $\frac{1}{10}$; $\frac{72}{7}$; $\frac{1}{10}$; $\frac{99}{16}$; $\frac{18}{1}$; $\frac{1}{8}$; $\frac{1}{6}$; $\frac{2}{1}$.

151–160. 13 s. 11 d. = 167 d. £$2\frac{7}{8} = \frac{23}{8} \times 20 \times 12$ = 690 d. $\frac{167}{690}$; $\frac{2220}{1}$; $\frac{27}{400}$; $\frac{500}{67}$; $\frac{2}{21}$; $\frac{19}{3360}$; $\frac{16}{3}$; $\frac{12}{1297}$; $\frac{32}{5}$; $\frac{451}{14}$.

161–170. 8 : 11; 3 : 5; 16 : 10; 2 : 4; 1 : 4; 3 : 8; 6 : 29; 31360 : 14; 5 : 36; 30 : 9.

171–180. 3 h. 15 m. = 195 m., and 19 dy. 7 h. 25 m. = 27805 m. $\frac{195}{27805} = \frac{39}{5561}$; $\frac{114}{5561}$; $\frac{576}{5561}$; $\frac{3252}{5561}$; $\frac{17}{5561}$; $\frac{8640}{5561}$; $\frac{5400}{5561}$; $\frac{126}{5561}$; $\frac{20448}{5561}$; $\frac{1638}{5561}$.

¶ 133. **181–190.** (p. 156.) 10 dy. is the unknown term, or the term for which an answer is required, and 5 dy. the given term. The ratio of 10 to 5 is $\frac{10}{5}$. Hence 100 $\times \frac{10}{5}$ = 200 m.; 800 m.; 40 m.; 220 m.; 190 m.; 174 m.; 155 m.; 241 m.; $17\frac{1}{2}$ m.; $18\frac{3}{4}$ m.

191–200. $\frac{36 \times 8}{6} = 48$; 72; 32; 144; 96; 24; 16; 18; 4; 8.

201–210. $\frac{\$14 \times 18}{45} =$ \$5.60; \$4.66; \$12.44; \$28; \$11.66; \$6; \$10; \$16; \$4.59; \$11.45.

211–220. $\frac{\$40 \times 6.5}{9} =$ \$28.89; \$23.33; \$87.78; \$212.72; \$16.39; \$45.56; \$36.28; \$2.40; \$360; \$7.09.

221–230. $\frac{£9 \times 100}{40} =$ £22.5, or £22 10 s.; £19 13 s. 9 d.; £8 2 s.; £6 8 s. $10\frac{1}{2}$ d.; £100; £22 $5\frac{1}{4}$ d.; £78 15 s.; £140 12 s. 6 d.; £22 2 s. $5\frac{3}{4}$ d.; £8 8 s. 9 d.

231–240. $\frac{90 \times 4.75}{1.20} = 356\frac{1}{4}$ bu.; $337\frac{1}{2}$ bu.; $393\frac{3}{4}$ bu.; 365 bu. 2 pk. 4 qt.; 440 bu. 2 pk. 4 qt.; $403\frac{1}{8}$ bu.; 450 bu.; $412\frac{1}{2}$ bu.; $431\frac{1}{4}$ bu.; 421 bu. 3 pk. 4 qt.

241–250. $\frac{5.5 \times 75}{3\frac{1}{3}}$ or $\frac{11 \times 3 \times 75}{2 \times 10} = 123\frac{3}{4}$ feet; $82\frac{1}{2}$ ft.; 165 ft.; 132 ft.; $103\frac{1}{8}$ ft.; $156\frac{3}{4}$ ft.; $138\frac{3}{16}$ ft.; 198 ft.; $144\frac{63}{80}$ ft.; 163.35 ft.

REDUCTION OF CURRENCIES.

To reduce Federal Money to the several Currencies.

Multiply the Sum by such a Fraction as \$1 *is of* £1.

\$1 = 4 s. 6 d., or 54 d. sterling. £1 = 240 d., and £$\frac{54}{240}$ = $\frac{9}{40}$, &c.

¶ **135.** **251–260.** (p. 157.) \$1 = £$\frac{9}{40}$ sterling, and $\frac{125 \times 9}{40}$ = £28$\frac{1}{8}$, or £28 2 s. 6 d. English or sterling money; \$1 = £$\frac{1}{4}$ Ca. Hence $\frac{125 \times 1}{4}$ = £31 5 s. Ca.; £37 10 s. N. E.; £50 N. Y.; £46 17 s. 6 d. Pa.; £29 3 s. 4 d. Ga.; £28 2 s. 6 d. N. E.; £37 10 s. N. Y.; £35 3 s. 1$\frac{1}{2}$ d. Pa.; £21 17 s. 6 d. Ga.

To reduce the several Currencies to Federal Money.

Reduce the Shillings, Pence, and Farthings, if any, to the Fraction of a Pound, and multiply the sum by such a fraction as £1 *is of* \$1.

261–270. 9 s. 8 d. = 111 d., or £$\frac{111}{240}$, or £0.4625. As £1 sterling = \$$\frac{40}{9}$, $\frac{£17.4625 \times 40}{9}$ = \$77.61; \$69.85; \$58.21; \$43.66; \$46.57; \$74.84; \$30.94; \$23.21; \$24.76; \$39.79.

271–280. £16 5 s. 1$\frac{1}{2}$ d. sterling; £18 1 s. 3 d. Ca.; £21 13 s. 6 d. N. E.; £28 18 s. N. Y.; £27 1 s. 10$\frac{1}{2}$ d. Pa.; £16 17 s. 2 d. Ga.; £1 4 s. 9 d. N. E.; £1 13 s. N. Y.; £1 10 s. 11$\frac{1}{4}$ d. Pa.; 19 s. 3 d. Ga.

281–290. (p. 158.) 7 s. 6 d. = 90 d. = £0.375, and $\frac{.375 \times 40}{9}$ = \$1$\frac{2}{3}$; \$1.50; \$1.25; \$0.93$\frac{3}{4}$; \$1; \$1.61; \$3.96; \$2.97; \$3.17; \$5.09.

291–300. £1 sterling = \$$\frac{40}{9}$, (¶ **134**,) and £1 Canada = \$4. Hence £1 Canada is $\frac{4}{\frac{40}{9}}$ of a pound sterling. And $\frac{4}{\frac{40}{9}} = \frac{4}{1} \times \frac{9}{40} = \frac{9}{10}$ Ans. for Canada currency.

As £1 N. E. = $$3\frac{1}{3}$, or $$\frac{10}{3}$, $\frac{\frac{10}{3}}{\frac{40}{9}} = \frac{3}{4}$ N. E.; $\frac{9}{16}$ N. Y.; $\frac{3}{5}$ Pa.; $\frac{27}{28}$ Ga.

296. By answer to example **291** *above*, £1 Canada is £$\frac{9}{10}$ sterling. Therefore (£9 2 s. 11 d.) × $\frac{9}{10}$ = £8 4 s. $7\frac{1}{2}$ d. Ans. for Canada currency; £6 17 s. $2\frac{1}{4}$ d. N. E.; £5 2 s. $10\frac{3}{4}$ d. N. Y.; £5 9 s. 9 d. Pa.; £8 16 s. $4\frac{1}{2}$ d. Ga.

¶ 137. 301–310. By ¶ **135**, $\frac{£7 \times 40}{9} = \$\frac{280}{9}$, the par value; but the par value of £1 + the premium = 1.09. Hence $\frac{280}{9} \times 1.09 = \$33.91$; $\frac{15 \times 40 \times 1.09}{9 \times 1} = \$72.67$; $116.27; 11 s. 6 d. = £$\frac{23}{40}$, &c. = $2.79; $2.26; $1.43; $63.64; $4.76; $45.78; $133.95.

311–320. By ¶ **135**, $\frac{\$100 \times 9}{40} = £\frac{900}{40}$, the par value. The par value of $1 + the premium = 1.08. Hence $\frac{900}{40} \div 1.08$, or $\frac{900}{40} \times \frac{1}{1.08}$ = £20 16 s. 8 d.; £145 16 s. 8 d.; £72 18 s. 4 d.; £1 8 s. $1\frac{1}{2}$ d.; £5 18 s. 9 d.; £4 $2\frac{1}{2}$ d.; £16 12 s. 6 d.; £7 1 s. $2\frac{1}{2}$ d.; £208 6 s. 8 d.; 5 s. $6\frac{1}{2}$ d.

321–330. (p. 159.) 3 s. 9 d. = £.1875, and $\frac{.1875 \times 40 \times 1.085}{9}$ = $0.90; $0.56; $1.37; $12.30; $44.12; $80.53; $4.58; $236.29; $$4822\frac{2}{9}$; $19.91.

331–340. $\frac{40 \times 9 \times 1}{40 \times 1.0975}$ = £8 4 s.; £1 9 s. $8\frac{3}{4}$ d.; £3 19 s. $5\frac{1}{4}$ d.; £5 14 s. $3\frac{1}{2}$ d.; £13 $10\frac{1}{2}$ d.; £29 4 s. $3\frac{1}{2}$ d.; 7 s. 2 d.; £1 19 s. $7\frac{1}{2}$ d.; £46 2 s. $6\frac{1}{2}$ d.; £164 $2\frac{1}{4}$ d.

PRACTICE, &c.

¶ 139. 341–350. $12\frac{1}{2}$ cts. = $$\frac{1}{8}$, and $14 \times \frac{1}{8}$ = $1.75; $2.33; $3.50; $$4.37\frac{1}{2}$; $2.625; $2.80; $5.25; $7; $4.66; $5.60.

351–360. (p. 160.) $25 ÷ $\frac{1}{8}$ 200 bu.; 150 bu.; 125 bu.; 100 bu.; 75 bu.; $62\frac{1}{2}$ bu.; 50 bu.; $66\frac{2}{3}$ bu.; 80 bu.; $133\frac{1}{3}$ bu.

361–370. $24; $36; $30; $27; $33; $45; $18; $21; $38.40; $28.80.

371–380. 50 ct. = $$\frac{1}{2}$, and $24 ÷ $\frac{1}{2}$ = 48 days; 32 dy.; 38.4 dy.; $42\frac{2}{3}$ dy.; $34\frac{10}{11}$ dy.; $25\frac{3}{5}$ dy.; 64 dy.; $54\frac{6}{7}$ dy.; 30 dy.; $27\frac{3}{7}$ dy.

381–390. $36 \times \frac{3}{8} = \$13.50$; \$18; \$27; \$31.50; \$22.50; \$20.25; \$21.60; \$29.25; \$28.80; \$24.75.

391–400. $\$27 \div \frac{1}{8} = 216$; 270; 162; 108; 135; 144; 72; $67\frac{1}{2}$; $86\frac{2}{5}$; 324.

¶ 140. COMPLEX ANALYSIS.

No rule affords the pupil a better opportunity for the exercise of his judgment, in performing the operations of examples, than Analysis, and especially Complex Analysis; and no rule affords the *teacher* a better opportunity for testing the thoroughness and independence of his pupils. In this work (the arithmetic) the section is an important one, embracing a large number of examples in many departments of business. The rule given on page 161, will apply to all examples, and needs no exceptions. Different teachers and pupils will, of course, have their favorite ways of varying and abbreviating it, after they have become familiar with it.

When the pupil shall have become expert in performing examples by the rule, step by step, it will be desirable that he shall receive some hints in regard to combining and varying the several steps of the process.

The following may be given as a specimen, which, as will be observed, is an extended application of ¶ **133.**

Write in the numerator of a fraction the term which is of the same kind as the required answer.

Take any two terms that are alike, (of the same denomination,) and consider whether, from their relation to the question, the answer will be greater or less than the numerator. Express the ratio of these two terms in the form of a fraction, making the greater number the numerator when the answer will be greater, and the less number the numerator when it will be less. Write this fraction as a multiplier after the numerator expressed.

Arrange the other terms in like manner, and reduce the fraction, &c.

The operation of Example **403**, p. 162, by the rule, (in the arithmetic,) and by the above variation, would be as follows: —

As 13 (men) corresponds to the required answer, we write that number as the numerator of a fraction, thus: $\frac{13}{\qquad}$. If 78 acres require 13 men, one acre will require $\frac{1}{78}$ as many. Hence 78 is written as a divisor, and the expression now becomes $\frac{13}{78}$——. If, to reap a field in 6 days, 13 men are required, 6 times as many men would be necessary to reap it in 1 day. Writing 6 as a multiplier, we have $\frac{13 \times 6}{78}$. Now it will require 72 times as many men to reap 72 acres, as it will 1 acre; therefore the expression becomes $\frac{13 \times 6 \times 72}{78}$. But it will require but $\frac{1}{36}$ as many men to reap a field in 36 days as it will to do it in 1 day. We must therefore divide by 36. Then $\frac{13 \times 6 \times 72}{78 \times 36} = 2$ Ans.

Writing, again, 13 as a numerator, we next take 78 acres and 72 acres, (both of the same kind,) the latter of which is the unknown term, or the term for which we wish to obtain an answer. Now it will require less men to reap 72 acres than to reap 78; that is, it will take $\frac{72}{78}$ as many. Writing this after the numerator, it becomes $\frac{13 \times 72}{78}$. Next, taking days, it will require less men to reap a field in 36 days than in 6 days; and our next multiplier will, therefore, be $\frac{6}{36}$, which, written after the former expression, gives $\frac{13 \times 72 \times 6}{78 \times 36} = 2$ Ans.

¶ 141. **403–412.** (p. 162.) *See above.* 2; 36; 4; 6; 24; 18; 8; 12; 9; 3.

413–423. $36 \times \frac{8}{6} = 48$; 144; 16; 32; 72; 24; 96; 12; 4; 9; 6.

424–434. $\frac{8 \times 36 \times 12}{6 \times 2} = 288$; 192; 64; 144; 96; 72; 36; 48; 32; 18; 24.

435–445. $\frac{\$3 \times 25 \times 16}{12 \times 5} = \$20$; \$5; \$2.50; \$12.50; \$18.75; \$8.75; \$7.50; \$10; \$1.25; \$6.25; \$16.25.

446–456. 2 tons $= 4480$ lb. $\frac{56 \times 900 \times 36}{4480 \times 2.50} = 162$ m.; 36 m.; $67\frac{1}{2}$ m.; 45 m.; 18 m.; $40\frac{1}{2}$ m.; $31\frac{1}{2}$ m.; 180 m.; $49\frac{1}{2}$ m.; $13\frac{1}{2}$ m.; 27 m.

457–467. $\frac{8 \times 900 \times 24}{1200 \times 16} = 9$ mo.; 2 mo.; 8 mo.; 6 mo.; 4 mo.; 1 mo.; 12 mo.; 24 mo.; 3 mo.; 16 mo.; 36 mo.

468. $\frac{8 \times 10 \times 100}{12\frac{1}{2} \times 40} = 16$. (*See* ¶ **76**, *and note to operation of Example* **57**, ¶ **129**.)

469. $\frac{1760 \times 6 \times 31 \times 3}{5 \times 2 \times 88} = 1116$ bu.

470–480. 54; 108; 432; 72; 144; 648; 324; 162; 216; 16; 48.

481. $\frac{\$14 \times 11 \times 17}{3 \times 45} = \$19.39$.

482–492. $\frac{18 \times 32 \times 16 \times 15 \times 5 \times 9 \times 3^{*}}{20 \times 12 \times 10 \times 6 \times 12 \times 12} = 9$; 36; 12; 27; 18; 54; 3; 4; 6; 2; 1.

493–504. £5 11 s. $7\frac{3}{4}$ d.; 64; 25; 80; 16; 20; 35; 112; 40; 15; 8; 1.

505–508. \$16.58; $337\frac{1}{2}$; \$198.86; 960.

509–519. (p. 164.) $9\frac{3}{5}$; $7\frac{1}{5}$; 12; 6; 18; 24; 16; $14\frac{2}{5}$; $10\frac{4}{5}$; $6\frac{2}{5}$; $8\frac{2}{5}$.

520–522. $\frac{\$495 \times 9 \times 13\frac{1}{3} \text{ mo.}}{7 \times 11\frac{1}{5} \text{ mo.}} = \$757.65$; 13; \$343.70.

523. $\frac{87.5 \times 3\frac{1}{3} \times \$27.75}{2\frac{7}{8} \times 15\frac{1}{3}}$, or $\frac{87.5 \times 8 \times 10 \times 27.75 \times 3}{23 \times 3 \times 1 \times 46} = 183\frac{318}{529}$ m.

524–525. 4; 96.

526. $66\frac{2}{3} = \frac{200}{3}$. $94\frac{2}{7} = \frac{660}{7}$. $\frac{10 \times 660 \times 3 \times 35 \times 40 \times 4 \times 10}{7 \times 200 \times 45 \times 50 \times 5.5 \times 8} = 8$ Ans.

527. $\frac{29.50 \times 21.5 \times 1.5 \times 8}{14.625 \times 7} = \$74.34$.

528. $\frac{300 \times 9 \times 6}{6 \quad 4} = 675$.

529. $\frac{18 \times 7 \times 6 \times 3.5 \times 16}{2 \times 2 \times 1} = 10584$ lb. $=$ 4 T. 14 cwt. 2 qr.

530. $\frac{7 \times 240 \times 6 \times 3 \times 32 \times 7}{40 \times 4 \times 2 \times 8 \times 9} = 294$.

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* This statement is reduced wholly by cancelling.

DUODECIMALS.

¶ 143. 531–540. (p. 166.) 22 7′; 39 6′ 3′′; 2 4′ 2′′ 9′′′; 4 2′ 9′′ 9′′′; 1′ 4′′ 11′′′ 3′′′′; 5′ 2′′ 1′′′ 3′′′′; 10 4′ 2′′ 6′′′; 20 8′ 5′′; 31 7′′ 6′′′; 15 9′ 1′′ 7′′′ 6′′′′.

541–550. 39 1′ 7′′ 8′′′; 2 6′ 4′′ 4′′′ 6′′′′; 1′ 9′′ 7′′′ 1′′′′ 4′′′′′; 12 2′ 5′′ 1′′′ 2′′′′; 7 9′ 1′′ 5′′′; 11 5′ 7′′ 10′′′; 6 6′ 7′′ 3′′′ 11′′′′; 1 11′ 3′′ 4′′′ 3′′′′; 1′ 1′′ 1′′′ 10′′′′ 9′′′′′; 3′′ 8′′′ 6′′′′ 5′′′′′.

551–554. 13 11′ × 1 9′ = 24 4′ 3′′, × 6 = 146 1′ 6′′. 1′ 6′′ = 18′′ or $\frac{18}{144}$ ft. = $\frac{1}{8}$. $146\frac{1}{8}$ Ans.; $231\frac{1}{4}$; $197\frac{7}{16}$; 585.

555. 128 × 8 9′ × 6 3′ = 7000 cu. ft., ÷ 128 = $54\frac{11}{16}$ cords. \$6.25 × $54\frac{11}{16}$ = \$341.80.

556–557. = 397 sq. ft. 4′, &c., ÷ 9 = 44 yd. 1 4′ = $44\frac{16}{108}$ yd. = $44\frac{4}{27}$ yd.; 78.

558. = 2940 cu. ft., ÷ 16 = $183\frac{3}{4}$.

559. = 50 yd. 7 10′ 6′′, and 7 10′ 6′′ = 1134′′. 1 sq. yd. = 1296.′′ $\frac{1134}{1296} = \frac{7}{8}$, and $50\frac{7}{8}$ yd. = Ans.*

						9	9′	9′′	9′′′	
560.						9	9′	9′′	9′′′	
					7′′	4′′′	4′′′′	3′′′′′	9′′′′′′	
				7	4	4	3	9		
			7	4	4	3	9			
			88	4	3	9				
			96	4′	7′′	9′′′	5′′′′	0′′′′′	9′′′′′′	
						9	9′	9′′	9′′′	
	6′	0′′	3′′′	5′′′′	10′′′′′	0′′′′′′	9′′′′′′′	6′′′′′′′′	9′′′′′′′′′	
6	0	3	5	10	0	9	6	9		
72	3	5	10	0	9	6	9			
867	5	10	0	9	6	9				
946	3′	7′′	8′′′	2′′′′	3′′′′′	2′′′′′′	1′′′′′′′	3′′′′′′′′	9′′′′′′′′′	Ans.

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* See Appendix, "Duodecimals."

¶ 145. **561.** 2 5′)17 6′ 3″(7 3′;
16 11
7′ 3″
7 3

562–565. 2 5′; 3′ 5″; 9 4′; 6′ 10″.
566–570. 1 7′ 6″; 9′ 9″; 4 4′; 8 8′; 2 2′.
571–575. 2 5′ 6″; 3 5′; 6 10′; 9′ 10″; 4 11′.
576–580. 6 1′; 8 2′; 4 1′; 12 2′; 4 8″.
581–585. 7″ 5‴; 12 10′; 1 2′ 10″; 2′ 5″ 8‴; 2′ 1″ 8‴.
582. 9 3′ 7″.

MISCELLANEOUS ANALYSIS.

¶ 146. **583.** (p. 168.) Father's age 35, and son's 5.

584. Both numbers are together equal to $6\frac{1}{2}$ times the less; therefore $13 \div 6\frac{1}{2} = 2$ the less, and $13 - 2 = 11$ the greater.

585. The two numbers are equal to $\frac{10}{7}$ $(1 + \frac{3}{7})$ times the greater. $25 \div \frac{10}{7} = 17\frac{1}{2}$ the greater. $25 - 17\frac{1}{2} = 7\frac{1}{2}$ the less.

586. 17 boys, and 34 girls.

587. Represent the first day's journey by 1. The second will then be 2, and the third $3 \times 2 = 6$, and their sum will be 10 times the first. $144 \div 10 = 14.4$ miles the first, 43.2 m. = the second, and 86.4 m. = the third.

588. The three contributed 6 times A.'s share, &c. A. $450, B. $900, and C. $1350.

589. The daughter has 1 share, the widow 1 share + 600, and the son 1 share + 600 + 800. Then 3 shares + $2000 are equal to $5000. $5000 — $2000 = 3000, ÷ 3 = $1000 for the daughter, widow $1600, son $2400.

590. Let 1 represent the first part, 2 the second, 2 + 7 the third, and 1 + 18 the fourth; in all 6 times the first part + 25. $194 - 25 = 169$, $\div 6 = 28\frac{1}{6}$ the first, &c., $56\frac{1}{3}$, $63\frac{1}{3}$, $46\frac{1}{6}$.

591. $6 \times 5\frac{1}{2} = 33$. $9 - 6 = 3$ m., the constable's gain per hour. $33 \div 3 = 11$ hours.

592. $(1 \times 22) + (8 \times 2) = 38$. $418 \div 38 = 11$ cows. $11 \times 2 = 22$ calves.

593. $\frac{127 + 13}{4} = 35$.

594–596. 30 ft.; 72; 84.

597. All together are equal to 4 times the first, $+ 8$. The numbers are 22, 24, 50.

598–599. $\frac{38\frac{3}{4}}{5\frac{2}{7} + 6\frac{1}{9} \text{ or } 11\frac{25}{63}} = 3\frac{1149}{2872}$ hours; $10\frac{10}{43}$ hours.

600–601. 5 boys, 10 men; the number $- \frac{3}{7} = \frac{4}{7}$. The remainder $- \frac{2}{9} = \frac{7}{9}$ of itself. $\frac{7}{9}$ of $\frac{4}{7} = \frac{4}{9}$. 18 is $\frac{4}{9}$ of the number, which is $40\frac{1}{2}$.

602. In one day, A. can do $\frac{1}{5}$ of $\frac{1}{2} = \frac{1}{10}$ of the work, B. $\frac{1}{4}$ of $\frac{1}{3} = \frac{1}{12}$, C. $\frac{1}{2}$ of $\frac{1}{6} = \frac{1}{12}$, and all together $\frac{16}{60}$. 1 (*the whole work*) $\div \frac{16}{60} = 3\frac{3}{4}$ days.

603–605. The difference between their ages would be 8 years, $= \frac{1}{2}$ the age of the elder, or the *entire* age of the younger. $8 + 5$ (or $16 - 3$) $= 13$ yr.; $16\frac{16}{29}$; $3\frac{3}{4}$ dy.

606. (p. 170.) Let the first flock be represented by 1, and then the others will be $1 - 33$, $1 - 95$, $1 - 200$, in all 4 times the first flock $- 328$. $668 + 328 = 996$, $\div 4 =$ 249 first flock, second 216, third 154, fourth 49.

607–608. 80; $320.

609. The first would fill it 3 times per hour, the second 4 times, and both together 7 times. The third will empty it 2 times per hour. $7 - 2 = 5$. It will therefore be filled 5 times per hour, or once in $\frac{1}{5}$ h., or 12 minutes.

610. $\frac{4}{9} - \frac{2}{6}$ of $\frac{4}{5} = \frac{8}{45}$. $32 \div \frac{8}{45} = 180$.

611. 48 seconds $= \frac{1}{75}$ hour. 12 miles $= 3840$ rods, $\div 75 = 51.2$, $+ 5 = 56.2$ rods, distance of the hare in advance of the hound when the latter starts. $18 - 12 = 6$ miles $= 1920$ r., the hounds gain per hour. $56.2 \div 1920 = \frac{56.2}{1920.0}$ h. $= 1$ min. $45\frac{3}{8}$ sec.

612–613. $8400; $3000.

614. $5 + 9 = 14$. $\frac{5}{14}$ of $196 = 70$, the first. $\frac{9}{14}$ of $196 = 126$, the second.

615. 11 — 8 = 3. $\frac{11}{3}$ of 19 = 69$\frac{2}{3}$, 1st. $\frac{8}{3}$ of 19 = 50$\frac{2}{3}$, 2d.

616–620. 6$\frac{46}{79}$; \$438.75; $\frac{46}{149}$, $\frac{43}{149}$, $\frac{60}{149}$; \$46, \$43, \$60; \$3.875.

621. His distance travelled, + "as far again" = 2 times his distance. Then 2 + $\frac{1}{13}$ + $\frac{2}{15}$ = 2$\frac{41}{195}$ times, &c. 119 ÷ 2$\frac{41}{195}$ = 53$\frac{362}{431}$ miles.

XI. PROPORTION.

¶ 151. 1–10. (p. 172.) 15; 28; 10; 9; 15; 1; 2; 10; 11; 27.

11–20. 1; 3.6; 9$\frac{1}{3}$; 1$\frac{1}{3}$; 2; 16; 8; 12$\frac{4}{7}$; 1.875; 15$\frac{7}{16}$.

21–30. 6$\frac{2}{3}$; 9$\frac{1}{3}$; 12$\frac{2}{3}$; 5$\frac{5}{6}$; 7$\frac{7}{16}$; 1.46875; 2$\frac{7}{13}$; 8.75; 7; 12.

31–40. $\frac{83}{875}$; 1$\frac{23}{27}$; 25; 4; $\frac{10}{21}$; 18; 7$\frac{3}{11}$; 6$\frac{3}{16}$; $\frac{1}{6}$; $\frac{1}{14}$.

41–50. 3.6; 1$\frac{5}{7}$; .6; 2$\frac{2}{7}$; 3.75; $\frac{1}{8}$; 18$\frac{1}{3}$; 4$\frac{1}{6}$; 2$\frac{11}{29}$; 46$\frac{2}{7}$.

¶ 152. 51–70. (p. 173.) 2 : 1 : : 7 : —; 3 : 2 : : 9 : —; 4 : 3 : : 8 : —, &c.

71–90. 4 : 3 : : 5 : —; 7 : 6 : : 8 : —, &c.

91–110. 17 : 16 : : 4 : —; 8 : 2 : : 3 : —, &c.

111–130. 4 : $\frac{5}{8}$: : 1 : —; 9 : $\frac{2}{7}$: : 1 : —, &c.

131–150. 3$\frac{1}{2}$; 6; 6; 12; 9$\frac{4}{9}$; 5$\frac{1}{5}$; 42$\frac{2}{3}$; 22$\frac{1}{2}$; 4$\frac{4}{11}$; 6; 11, &c.

¶ 155. 152. (p. 175.) 20 : 44 : : 5 : ? — that is, $\frac{44 \times 5}{20}$ = 11 Ans.

153–160. 5; 20 tons; 10 dy.; 7 men; 14 dy.; 5; 430$\frac{119}{273}$ gall.; 11$\frac{16}{49}$ min.

161–170. \$3.69; \$3.375; \$98.33; \$171.875; 6 per cent.; 1 yr. 7 mo. 6 dy.; 46$\frac{1}{2}$ per cent.; 5 yr. 9 mo. 18 dy.; 117; \$40.

171–177. 40; 70 bu. 4$\frac{2}{3}$ qt.; 3 h.; $\frac{3}{4}$: 1.5 : : 9$\frac{3}{4}$: 19$\frac{1}{2}$; 36; 10 dy.; $\frac{7}{8}$: 3$\frac{1}{2}$: : 4$\frac{1}{2}$: 18 yd.

178–184. \$36.96; £9 11 s. 8 d. = 2300 d., and 4 s. 6 d. = 54 d. 54 : 2300 : : 1 : \$42$\frac{16}{27}$; 144; \$57.02; 36 oz. 15 dwt.; 128 ft. 10.875 in.; 100.

185–190. (p. 177.) 4 mo. 5 dy.; 10.5; 12; £$1\frac{3}{16}$ = £$\frac{19}{16}$. Then 112 : 24.5 : : £$\frac{19}{16}$: 5 s. $2\frac{1}{4}$ d.; $1530.675; $$\frac{7}{30}$, or 23 cts. +.

COMPOUND PROPORTION.

¶ 157. 191. (p. 178.)

$$\begin{array}{l}9:13\\7:17\end{array} :: 63 : 221 \text{ rods Ans.}$$

By Analysis.

$$\frac{63 \times 13 \times 17}{9 \times 7} = 221 \text{ Ans.}$$

192–193. 5 men; $18\frac{1}{2}$ reams.

194.

$$\begin{array}{rcl}60 & : & 15\\45 & : & 100\\8 & : & 9\end{array} :: 16 : 8\tfrac{7}{16} \text{ dy. Ans., or thus:}$$

$$\begin{array}{rcl}4 & : & 3\\8 & : & 9\end{array} \qquad \frac{16 \times 15 \times 100 \times 9 \times 3 \times 9}{60 \times 45 \times 8 \times 4 \times 8} = 8\tfrac{7}{16}.$$

195–196. $142.50; 9 men.

197. 8 mo. 24 dy. = 8.8 mo.

$$\begin{array}{rcl}\$1.00 & : & \$496.50\\12 \text{ mo.} & : & 8.8 \text{ mo.}\end{array} :: .06 : \$21.846 \text{ Ans.}$$

198.

$$\begin{array}{rcl}496.50 & : & 1.00\\8.8 & : & 12\end{array} :: 21.846 : 6 \text{ per cent.}$$

199.

$$\begin{array}{rcl}.06 & : & 21.846\\8.8 & : & 12\end{array} :: 1.00 : \$496.50.$$

200.

$$\begin{array}{rcl}.06 & : & 21.846\\496.50 & : & 1.00\end{array} :: 12 : 8.8 \text{ mo.}$$

ARBITRATION OF EXCHANGE.

¶ 159. 201. (p. 180.)

$$\begin{array}{lcl}25 \text{ copecks} & : & 4 \text{ pence}\\16 \text{ pence} & : & 5 \text{ piasters}\\12 \text{ piasters} & : & 1 \text{ florin}\\7 \text{ florins} & : & 36 \text{ francs}\end{array} :: 3000 \text{ copecks} : 64\tfrac{2}{7} \text{ francs Ans.}$$

202.

$$\begin{array}{rcr}8 & : & 30\\14 & : & 3\\6 & : & 5\\10 & : & 126\end{array} :: 24 : 202\tfrac{1}{2} \text{ pounds Ans.}$$

203–205. 6400 roubles ; 15 bags ; 60 yds.

206. (p. 181.)

$$\left.\begin{array}{r} 104 : 100 \\ 70 : 100 \\ 142 : 100 \\ 105 : 100 \end{array}\right\} : : 6461 : 5952\tfrac{8}{21} \text{ Ans.}$$

207–210. \$1.08 ; 2975 reas ; £416 13 s. 4 d. ; 13 bbls.

FELLOWSHIP.

¶ 161. **211.** (p. 182.) \$200 + \$500 = \$700 the whole stock. Then

700 : 200 : : 350 : \$100 A.'s share.
700 : 500 : : 350 : \$250 B.'s share.

The same by analysis; $\frac{200}{700} \times 350 = 100$ A.'s share. $\frac{500}{700} \times 350 = 250$ B.'s share.

212. A. \$58.50, B. \$45.50, C. \$65.

213. A. \$156, B. \$312, C. \$468.

214. 70, 42, 56, 28.

215. A. \$24, B. \$12, C. \$36.

216. A. \$56.19, B. \$233.33 ; C. \$210.48.

217–218. \$$\frac{53}{78}$ or 68 cts. ; A. \$160, B. \$80, C. \$40.

219. Widow \$400, eldest son \$480, youngest son \$400, daughter \$320.

220. $\frac{1}{2}, \frac{1}{3}, \frac{3}{4}, \frac{7}{12} = \frac{6}{12}, \frac{4}{12}, \frac{9}{12}, \frac{7}{12}$. Fractions of a common denominator are to each other as their numerators. Hence, use the above numerators as "shares." 150, 100, 225, 175, Ans.

COMPOUND FELLOWSHIP.

¶ 163. **221.** (p. 183.)

$$\begin{array}{rl} 200 \times 4 = & 800 \qquad 1800 : 800 : : 500 : \$222.22 \text{ A.'s.} \\ 500 \times 2 = & \underline{1000} \qquad 1800 : 1000 : : 500 : \$277.78 \text{ B.'s.} \\ & 1800 \end{array}$$

222. A. \$100, B. \$96, C. \$80.

223. A. \$78, B. \$93.60, C. \$62.40.

224. Let the "certain sum," and "certain time," of which fractional parts are to be taken, be represented by a whole number, or 1. Then

$1 \times 1 = 1$

$2 \times \frac{1}{3} = \frac{2}{3}$

$3 \times \frac{2}{5} = \frac{6}{5}$

$\overline{2\frac{13}{15}}$

$2\frac{13}{15} : 1 :: 559 : \$195$ A.'s.

$2\frac{13}{15} : \frac{2}{3} :: 559 : \$130$ B.'s. Ans.

$2\frac{13}{15} : \frac{6}{5} :: 559 : \$234$ C.'s.

225. (p. 184.)

800 × 12 mo. = 9600

1000 × 9 mo. = 9000

1500 × 5 mo. = 7500

$\overline{26100}$

26100 : 9600 : : 1592.10 : $585.60 A.

26100 : 9000 : : 1592.10 : $549. B.

26100 : 7500 : : 1592.10 : $457.50 C.

226. A. $52.50, B. $17.50, C. $21, D. $63, E. $140.

227. A. $10.91, B. $13.64, C. $15.45.

228. A. $1.80, B. $1.92, C. $2.

229.

300 × .60 = 180.00

400 × .50 = 200.00

600 × .45 = 270.00

$\overline{650.}$

650 : 180 : : 162.50 : $45. A.'s share.

650 : 200 : : 162.50 : $50. B.'s "

650 : 270 : : 162.50 : $67.50 C.'s "

230. A. $\$9\frac{3}{8}$, B. $\$11\frac{1}{4}$, C. $\$6\frac{7}{8}$, D. $\$13\frac{1}{8}$.

XII. AVERAGE.

¶ 165. 1–10. (p. 185.) 2; 5; 5; 4; 5; 6; 8; 3; 5; 15.

11–20. 15; 62; 73; 17; 40; 14; 20; 10; 9; 10.

21–30. $16\frac{1}{2}$; $29\frac{1}{2}$; $27\frac{1}{2}$; $89\frac{1}{2}$; $32\frac{2}{3}$; $25\frac{1}{3}$; $27\frac{2}{3}$; $3\frac{3}{4}$; $22\frac{3}{4}$; $26\frac{1}{2}$.

31–40. $2\frac{5}{16}$; $11\frac{7}{12}$; $6\frac{5}{8}$; $4\frac{3}{8}$; $12\frac{5}{16}$; 8.53; $35\frac{413}{600}$; $1\frac{77}{240}$; 265.5; $56\frac{1}{4}$.

41–45. $31\frac{3}{4}$ yd.; $173\frac{3}{4}$ lb.; 61 tons of hay, 35 bu. of grain; $\$1.17\frac{1}{2}$ per bu.; 12 yr. 7 mo. 6 dy.

ALLIGATION MEDIAL.

¶ 167. 46–50. (p. 187.) 30 cts.; $22\frac{3}{8}$ carats; 10 oz.; $71\frac{5}{12}°$; 7 cts.

EQUATION OF PAYMENTS.

¶ 169. 51–54. (p. 188.) 4 mo. 5 dy.; 80 dy.; 18 mo. 4 dy.; 49 dy.; March 10.

55. (p. 189.) The first payment is due in 0 days, and the other three respectively, in 30, 57, and 72 days, from March 4. Then

200
150 × 30 = 4500
200 × 57 = 11400
300 × 72 = 21600
850. 37500

37500 ÷ 850 = 44 days. 44 days added to March 4 = April 17, Ans.

56. The last three payments are due in 11, 23, and 35 dy. Sum of products 26500. Sum of payments $1400. 26500 ÷ 1400 = 19 dy., or Jan. 26. The account was on interest from Jan. 26 to Oct. 9, (8 mo. 13 dy.) Interest on $1400 for 8 mo. 13 dy., = $59.03.

57. Payments due in 0, 5, 6, and 22 dy. Sum of products 10100. Sum of payments $1000. 10100 ÷ 1000 = 10 dy., or March 29. Interest on $1000 from March 29 to Dec. 11, (8 mo. 12 dy.,) $42.

58. Times, 0, 9, 21, and 23 dy. Sum of products 987.50 Sum of payment $61.75. Due in 16 dy., or Oct. 26.

59. Times, 0, 19, and 61 dy. Products 3170.58. Payments $150.71. Due in 21 dy., or June 26.

60–68. In 19 dy.; May 29; Jan. 24; Jan. 20; Feb. 18; May 22; Sept. 4; April 5; May 12.

ALLIGATION ALTERNATE.

¶ 171. 69–75. (p. 191.)

Assume 1 lb. × 5 ct. costs 5 ct.
" 1 lb. × 8 ct. costs 8 ct.
2 lb. cost 13 ct.
2 lb. at 7 ct. cost 14 ct.
14 — 13 = 1 deficiency.

As the assumed quantities cost too little, we must diminish the quantity whose price is below the price of the mixture, or increase the one above it. The difference between the price of the mixture, 7, and the price greater, 8, is 1. This *difference*, 1, is continued in the deficiency, 1, *once*. Hence we may add 1 to the quantity at 8 cts., = 1 at 5 ct., 2 at 8 ct., Answer for the mixture at 7 cts.

Proof: 1 lb. at 5 ct. costs 5 ct.
2 lb. at 8 ct. cost 16 ct.
3 lb. cost 21 ct.
3 lb. at 7 ct. cost 21 ct.

At 6 ct., assume 2 lb. each. Then

2 × 5 = 10
2 × 8 = 16
4 lb. cost 26 ct.
4 lb. at 6 ct. = 24 ct.
Excess 2 ct.

8 — 6 = 2. 2 ÷ 2 = 1. Take 1 lb. less at 8 ct., = 2 lb. at 5 ct. and 1 lb. at 8 ct. Ans.

At 5½ ct., assume 5 lb. each. Excess 10 ct. Take 4 lb. less at 8 ct., = 5 at 5 ct., and 1 at 8 ct.

At 6½ ct., take 2 lb. at 5 ct. and 1 lb. at 8 ct. Deficiency 1½ ct. Take 1 less at 5 ct., = 1 at 5 ct and 1 at 8 ct.; 1 at 5 ct., 5 at 8 ct.; 5 at 5 ct., 7 at 8 ct.; 1 at 5 ct., 3 at 8 ct.

76–80. Assume 2 at .70, and 3 at 1.00. Excess .40. Take 2 gall. less at 1.00, = 2 gall. at .70, 1 gall. at 1.00; 1 at .70, 2 at 1.00; 5 at .70, 1 at 1.00; 1 at .70, 5 at 1.00; 1 of each.

81–85. Assume 2 bu. each. Excess 20 ct. Take 1 less at .70, or 2 more at .40, = 2 at .40 and 1 at .70, or 4 at .40 and 2 at .70; 5 at .40 and 1 at .70; 1 at .40, 2 at .70; 1 at .40, 5 at .70; 1 of each.

86–90. Assume 5 lb. each. Excess 25 ct. Take 1 less at .75, = 5 at .30, 4 at .75; for 62½ ct. assume 5 at .30 and 10 at .75. Deficiency .37½. Take 3 lb. more at .75, = 5 at .30 and 13 at .75; 5 at .30 and 1 at .75; 2 at .30 and 1 at .75; 7 at .30 and 2 at .75.

91–95. Assume 1 oz. of each. Excess 2 cts. Take ½ oz. more of 12 carats, or ½ oz. less of 20 carats, = 1½ of 12, 1 of 18, and 1 of 20, &c.

For 15 carats fine, assume 1 oz. each. Excess 5 ct. Add 1⅔ of 12, = 2⅔ of 12, 1 of 18, and 1 of 20. Or we may assume 2 oz. each. Excess 10 ct. Add $3\frac{1}{3}$ of 12, = $5\frac{1}{3}$ of 12, 2 of 18, and 2 of 20.

For 17 carats, assume 1 oz. each. Deficiency of 1 ct. Add 1 oz. of 18, = 1 of 12, 2 of 18, and 1 of 20; or we may assume 1 oz. of 12, 2 of 18, and 3 of 20. Excess 6 ct. Add $1\frac{1}{6}$ of 12, &c.; or assume 2 oz. each. Deficiency 2 ct. Add 2 oz. of 18 carats, &c.

For a mixture of 19 carats fine, assume 1 oz. each. Deficiency 7 ct. Add 7 oz. of 20, = 1 of 12, 1 of 18, and 8 of 20; or we may assume 5 oz. each. Deficiency 35 ct. Add 35 oz. of 20, &c.

For 14 carats, assume 1 oz. each. Excess 8 ct. Add 4 oz. of 12, = 5 of 12, 1 of 18, and 1 of 20; or assume 1 of 12, 2 of 18, and 3 of 20, and add 12 oz. of 12 carats, &c.

96–100. (p. 192.) Assume 1 gall. each. Deficiency 21 ct. Add 7 gall. at 25 ct., = 1 gall. at .20, 8 gall. at .25, and 1 gall. of water.

For a mixture at 18 ct. per gall., assume 2 gall. each. Deficiency 18 ct. Add 9 gall. at .20, (or take 1 gall. less of water,) = 11 gall. at 20, 2 at 25, and 2 of water.

For 16 ct., assume 1 gall. each. Deficiency 3 ct. Add $\frac{3}{4}$ gall. at 20 ct., = $1\frac{3}{4}$ gall. at .20, 1 at .25, and 1 of water.

For 21 ct., assume 1 gall. at .20, 2 gall. at .25, and 3 gall. of water. Deficiency 56 ct. Add 14 gall. at .25, = 1 gall. at .20, 16 gall. at .25, and 3 gall. of water. Or we may take $2\frac{2}{3}$ gall. *less* of water, &c.

For $12\frac{1}{2}$ ct., assume 5 gall. each. Excess .$37\frac{1}{2}$. Take 3 gall. less at .25, or add 3 gall. of water, = 5 gall. at .20, 2 at .25, and 5 of water, &c.

101–105. 101 performed.

For a mixture at 20 ct., assume 10 gall. of water.* Excess $2.00. Add 10 gall., = 20 gall.

For 22 ct., assume 5 gall. of water. Excess $1.90. Add $8\frac{7}{11}$ gall., = $13\frac{7}{11}$ gall.

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* Quantities already given (as 50 gall. vinegar) must not be varied.

For $18\frac{3}{4}$ ct., assume 20 gall. Excess .375. Add $4\frac{2}{3}$ gall., $= 24\frac{2}{3}$ gall.

For 16 ct., assume 25 gall. Excess $2.00. Add $12\frac{1}{2}$ gall., $= 37\frac{1}{2}$ gall.

106–110. Assume 10 gall. of water. Excess $1.25. Add $3\frac{1}{3}$ gall., $= 13\frac{1}{3}$ gall.

For 25 ct., assume 30 gall. Excess $2.50. Add 10 gall., = 40 gall.

For $12\frac{1}{2}$ ct., assume 100 gall. Excess $2.50. Add 20 gall., = 120 gall.

For $33\frac{1}{3}$ ct., assume 5 gall. Excess $5.00. Add 15 gall., = 20 gall.

For 30 ct., assume 25 gall. Excess .50. Add $1\frac{2}{3}$ gall., $= 26\frac{2}{3}$ gall.

111–115. Assume 10 gall. each, at .32 ct. and of water. Excess 2.35. Add $11\frac{3}{4}$ gall. of water, $= 21\frac{3}{4}$ gall.

For a mixture at 28 ct. per gall., assume 15 gall. at 32 ct. and 5 of water. Deficiency 5 ct. Add $1\frac{1}{4}$ gall. 32 ct., $= 16\frac{1}{4}$ gall. at .32 and 5 of water.

For 19 ct., assume 10 gall. each. Excess 2.85. Add 15 gall. water, = 10 at .32 and 25 of water.

For 15 ct., assume 20 gall. each. Excess 5.05. Add $33\frac{2}{3}$ gall. water, = 20 gall. at 32 and $53\frac{2}{3}$ of water.

For 23 ct., assume 5 gall. each. Excess $1.55. Add $6\frac{17}{23}$ gall. of water, = 5 gall. at .32 and $11\frac{17}{23}$ gall. of water.

116–120. Assume 1 lb. at 10 ct., 2 at 18 ct., and 3 at 20 ct. Deficiency 8 ct. Add 8 lb. at 20 ct., making 1 lb. at 10 ct., 2 at 18 ct., and 11 at 20 ct., in all a mixture of 14 lb. By ¶ **160**, as 14 lb., the mixture obtained, is to 200 lb., the mixture required, so are the several proportional quantities obtained, 1 lb., 2 lb., and 11 lb., to the several proportional quantities required. Thus: —

14 : 200 : : 1 : $14\frac{2}{7}$ lb. at 10 ct.
14 : 200 : : 2 : $28\frac{4}{7}$ lb. at 18 ct.
14 : 200 : : 11 : $157\frac{1}{7}$ lb. at 20 ct.
200 lb. the quantity required.

For 14 ct., assume 1 lb. of each. Excess 6 ct. Add $1\frac{1}{2}$ lb. at 10 ct., = $2\frac{1}{2}$ lb. at .10, 1 at .18 and .20 each. In all $4\frac{1}{2}$ lb. Then 4.5 : 200 : : 2.5, &c., = $111\frac{1}{9}$ lb. at 10 ct., $44\frac{4}{9}$ at .18 and .20 each.

For 17 ct., assume 10 lb. of each, and balance the deficiency by adding 10 lb. at .20, &c., &c., = 50 lb. at .10 and .18 each, and 100 at .20.

For 16 ct., assume 1 lb. of each, &c., = $66\frac{2}{3}$ lb. of each kind.

For 12 ct., assume 6 lb. at .10, and 1 at .18 and .20 each. Add 1 lb. at .10, &c., = $155\frac{5}{9}$ at .10, $22\frac{2}{9}$ at .18 and .20 each.

121–125. Assume 1 lb. at .75, 2 at $.87\frac{1}{2}$, and 2 at 1.00. In all 5 lb. Then 5 : 150 : : 1, &c., &c., = 30 lb. at .75, 60 lb. at $.87\frac{1}{2}$, and 60 lb. at 1.00.

For 85 ct., 70 lb., 40 lb., and 40 lb. Ans.

For 95 ct., 20 lb., 20 lb., 110 lb.

For 80 ct., 110 lb., 20 lb., 20 lb.

For 78 ct., 120 lb., 24 lb., 6 lb.

126–130. Assume 30 gall. Excess \$1.50. Add 30 gall., = 60 gall.; 120 gall.; 15 gall.; 6 gall.; $34\frac{2}{7}$ gall.

XIII. INVOLUTION AND EVOLUTION.

A table of Roots and Powers, convenient for reference, will be found in the Appendix.

¶ 173. 1–10. (p. 193.) 256; 64; 16; 1; .125; 1.5625; .00004096; 19.683; 130321; 13.0321.

11–50. The powers of 2 = 4, 8, 16, 32. Powers of 3 = 9, 27, 81, 243. Powers of $\frac{2}{3} = \frac{4}{9}, \frac{8}{27}, \frac{16}{81}, \frac{32}{243}$. Powers of $1\frac{1}{2}$ or $\frac{3}{2} = \frac{9}{4}$ or $2\frac{1}{4}$, $\frac{27}{8}$ or $3\frac{3}{8}$, $\frac{81}{16}$ or $5\frac{1}{16}$, $\frac{243}{32}$ or $7\frac{19}{32}$. Powers of 2.5 = 6.25, 15.625, 39.0625, 97.65625. Powers of 40 = 1600, 64000, 2560000, 102400000. Powers of $3\frac{1}{3}$ or $\frac{10}{3} = \frac{100}{9}$ or $11\frac{1}{9}$, $\frac{1000}{27}$ or $37\frac{1}{27}$, $\frac{10000}{81}$ or $123\frac{37}{81}$, $\frac{100000}{243}$ or $411\frac{127}{243}$. Powers of $2\frac{3}{4}$ or $\frac{11}{4} = \frac{121}{16}, \frac{1331}{64}, \frac{14641}{256}, \frac{161051}{1024}$. Powers of $\frac{3}{10} = \frac{9}{100}, \frac{27}{1000}, \frac{81}{10000}, \frac{243}{100000}$. Powers of .09 = .0081, .000729, .00006561, .0000059049.

¶ 175. 61–70. (p. 194.) 2; 3; 9; 4; 7; 5; 6; 1; 8; 10.

71–80. 2; 5; 3; 7; 10; 4; 8; 6; 9; 1.

SQUARE ROOT.

¶ 179. 81–100. (p. 197.) 11; 15; 20; 19; 12; 17; 14; 16; 13; 18; 25; 30; 21; 27; 22; 29; 23; 24; 26; 28.

101–120. 3.1; .32; 3.7; 40; 50; 3.3; .39; 36; 3.4; 38; $20\frac{1}{4} = \frac{81}{4}$, and $\sqrt{\frac{81}{4}} = \frac{9}{2}$ or $4\frac{1}{2}$;* 41; 4.9; .43; 4.7; 42; 44; 4.6; 4.8; $12\frac{1}{4} = \frac{49}{4}$, and $\sqrt{\frac{49}{4}} = \frac{7}{2}$ or $3\frac{1}{2}$.*

121–140. $\frac{2}{3}$; $\frac{4}{5}$; $\frac{1}{2}$; $\frac{3}{7}$; $\frac{8}{3}$; $\frac{8}{3}$ or $2\frac{2}{3}$; $2\frac{1}{2}$; $\frac{9}{12}$; $3\frac{2}{3}$; $3\frac{1}{4}$; $2\frac{2}{5}$; $3\frac{1}{6}$; $1\frac{5}{7}$; $2\frac{1}{8}$; $8\frac{1}{3}$; $15\frac{1}{2}$; $5\frac{5}{7}$; $\frac{1}{17}$; $\frac{9}{48}$; $12\frac{6}{7}$.

141–160. 1.41; 2.64; 3.60; 5.38; 2.40; 7.93; 11.83; 2.76; 9.97; 24.49; 4.89; 1.03; 21.93; 6.14; 19.41; .61; 3.81; 6.63; 13.26; 5.47.

161–170. 12 r.; 13 r.; 9 r.; 41 r.; 4.8 r.; 14 r.; $3\frac{1}{2}$ r.; $8\frac{1}{3}$ r.; $15\frac{1}{2}$ r.; 38 r.

171–180. 1.49; 2.449; .945; 269.91; 50.7; 106.67; 999; 9.949; 1230; 8072.

181–200. .547; $\frac{1}{5}$* = .2, and $\sqrt{.200000} = .447$; .894; 1.095; 1.341; 1.643; .353; .707; 1.060; .176; .530; .117; .235; .471; .935; .589; .942; 1.178; .883; .824.

¶ 180. CUBE ROOT.

The method of explaining the rule for Cube Root by means of blocks, so shaped as to exhibit a cube with successive additions in sections, is a method usually quite satisfactory to pupils, being more appreciable to the eye than a purely mental process. It is very desirable to have some such helps to render the reasons of the rule clear and intelligible, but they must, nevertheless, be regarded as helps *only*, and must not be allowed to supersede some such explanation as is given on pp. 198, 199, which is based upon the principles of pure numbers.

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* The roots of fractions are obtained by extracting the root of the numerator, and of the denominator, separately. Mixed numbers must be reduced to improper fractions before extracting, and fractions of which the terms are not exact powers must be reduced to a decimal, &c.

Illustration of the Rule. (¶ **181.**)

Required the cube root of 12167.

$$\begin{array}{rrr} & & 1\dot{2}16\dot{7}\ (23\ \text{Ans.} \\ & 2^3 = & \underline{8\quad} \\ \text{Trial Divisor.} & & \\ 2 \times 2 \times 3 = & 12\text{-}09^{*}) & 4167 \\ 30 \times 2 \times 3 = & \underline{180} & \underline{4167} \\ \text{Complete Divisor.} & 1389 & \end{array}$$

In the above example, after pointing off the number into periods, we find by trial that the largest cube in the left hand period is 8, and after placing its root, 2, at the right, we subtract the cube (2 × 2 × 2 = 8) from the first period, and to the remainder, 4, bring down the next period, 167, for a dividend. We next take three times the square of the root figure already found for a trial divisor, thus: $2^2 \times 3$, or 2 × 2 × 3 = 12. This divisor is contained in the hundreds (41) of the dividend 3 times, which we place in the root, and also its square, 9, at the right of the trial divisor, supplying tens' place with a cipher. Then multiplying 30 times the root already found, 2, by the last root figure, 3, (30 × 2 × 3 = 180,) we place the product, 180, under the trial divisor, to which it is added, forming a complete divisor, 1389. Multiplying this divisor by the last root figure, 3, we obtain the product, 4167, which, subtracted from the dividend, leaves no remainder.

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* The square of the second root figure, 3, with tens' place filled by a cypher.

Example 201. (p. 201.)

$$\begin{array}{rl} & \dot{1}33\dot{1}\,(11\text{ Ans.}\\ 1^3 = & 1\\ \hline 1^2\times 3 = 3\text{-}01) & 331\\ 30\times 1\times 1 = \quad 30 & 331\\ \hline & 331 \end{array}$$

Ex. 205.

$$\begin{array}{rl} & \dot{6}85\dot{9}\,(19\\ 1^3 = & 1\\ \hline 1^2\times 3 = 3\text{-}81) & 5859\\ 30\times 1\times 9 = 270) & 5859\\ \hline & 651 \end{array}$$

¶ **181.** **201–220.** (p. 201.) *See above.* 11; 20; 30; 12; 19; 40; 13; 18; 50; 14; 17; 21; 16; 22; 15; 23; 29; 24; 31; 28.

221–240. (p. 202.) 25; 33; 3.7; 41; 26; 35; 4.4; 46; 27; .48; 4.2; 39; 32; 4.9; 36; .45; 38; .47; 4.3; .34.

241–260. $\frac{2}{3}$; $\frac{1}{4}$; $\frac{5}{6}$; $42\frac{7}{8} = \frac{343}{8}$, and $\sqrt[3]{\frac{343}{8}} = \frac{7}{2}$ or $3\frac{1}{2}$; $2\frac{2}{3}$; $\frac{5}{9}$; $\frac{1}{10}$; $2\frac{3}{4}$; $\frac{9}{20}$; $2\frac{2}{5}$; $\frac{2}{30}$; $1\frac{1}{5}$; $4\frac{1}{2}$; $5\frac{1}{2}$; $4\frac{1}{3}$; $2\frac{4}{5}$; $6\frac{2}{3}$; $10\frac{1}{2}$; $9\frac{1}{2}$; $4\frac{1}{5}$.

261–280. 1.25; 1.91; 2.35; 3.07; 1.79; 3.97; 5.19; 1.97; 4.63; 8.43; 2.88; 1.02; 7.83; 3.35; 7.22; .72; 2.22; 4.44; 8.89; 3.10.

281–290. 12 in.; 11 in.; 19 in.; $10\frac{1}{2}$ in.; $4\frac{1}{2}$ in.; 2.8 in.; 43 in., or 3 ft. 7 in.; 4 ft. 1 in.; $4\frac{1}{5}$ in.; 51 in. = 4 ft. 3 in.

291–300. 13.2; 2.3513; 8300; 9.27; 3.6593; 4.0207; .4344; .9026; 10.09; 7.806; 9.84.

301–320. .952; .793; .634; .476; .317; .158; .119; .111; .170; .341; .512; .683; .854; $\frac{1}{8} = .125$, and $\sqrt[3]{.125} = .5$; .793; 1.040; 1.5; 2; $2\frac{1}{2}$; 35.

XIV. PROGRESSION, OR SERIES.

¶ **185.** **1–10.** (p. 203.) 1, 3, 5, 7, 9, 11, 13, 15, 17; —5, 8, 11, 14, 17, 20, 23, 26, 29;—2, 6, 10, 14, 18, 22, 26, 30, 34; — 2, $2\frac{1}{2}$, 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, 5, $5\frac{1}{2}$, 6, &c.

11–30. 95, 90, 85, 80, 75; 90, 80, 70, 60, 50, &c.

31–50. 55, 60, 65, 70— 80, 85, 90, 95, 100; 63, 66, 69, 72 — 78, 81, 84, 87, 90, &c.

In performing examples in Progression, it frequently aids the pupil to express, by means of letters and signs, the different steps of the process, and then to substitute the value of the letters, &c. Such an expression is called a **FORMULA.**

Let $a =$ the first term, $d =$ common difference, $l =$ last term, $s =$ sum of terms, and $n =$ the number of terms. The formula for finding the last term, according to the rule on page 204, will then be as follows: $l = a + (n - 1)\,d$. For a decreasing series it will be, $l = a - (n - 1)\,d$. For the common difference, $d = \frac{l - a}{n - 1}$ &c., and for the sum of the series, $s = \frac{a + l}{2} \times n$, &c.

¶ 187. **51.** (p. 204.) Formula, $l = a + (n - 1)\,d$; that is, $5 + (11 - 1)\,2 = 25$, last term. For the sum, $s = \frac{a + l}{2} \times n$, and $\frac{5 + 25}{2} \times 11 = 165$, sum of terms.

52–56. 120; 6, 9th t., 162, sum; 78; $4\frac{1}{2}$ l. t., 87 s.; $3140.

¶ 189. **57.** $d = \frac{l - a}{n - 1}$ or $\frac{25 - 5}{11 - 1} = 2$ com. diff., 165 sum.

58–62. 1, 120; 3, 162; 78, 1; 87, $\frac{1}{2}$; 6 per cent.

¶ 191. **63.** $\frac{25 - 5}{2} + 1 = 11$, number of terms, 165 sum.

64–68. 15, 120; 9, 162; 12, 78; 12, 87; 20 y., $3140.

¶ 193. **69.** (p. 207.) $\frac{2 \times 165}{5 + 25} = 11$ n. t. 2 com. diff.

70–74. 15, 1; 9, 3; 12, 1; 12, $\frac{1}{2}$; 20 y., 6 per cent.

¶ 195. **75.** (p. 208.) $\frac{2 \times 165}{11} - 5 = 25$ last t., 2 com. diff.

76–80. 1, 1; 30, 3; 1, 1; $4\frac{1}{2}$, $\frac{1}{2}$; $100, 6 per cent.

GEOMETRICAL PROGRESSION.

¶ 196. **81–90.** (p. 209.) 1, 2, 4, 8, 16, 32, 64, 128, 256; — 2, 6, 18, 54, 162, 486, 1458, 4374, 13122, &c.

91–110. 210, 105, $52\frac{1}{2}$, $26\frac{1}{4}$, $13\frac{1}{8}$; 140, $46\frac{2}{3}$, $15\frac{5}{9}$, $5\frac{5}{27}$, $1\frac{59}{81}$; &c.

111–130. 90, 180, — 720, 1440, 2880, 5760, 11520; 40, 120, — 1080; 3240, 9720, 29160, 87480; &c.

¶ 198. In the formula, r will represent *ratio.* Then $l = r^{n-1} \times a$.

131–135. (page 210.) $2^6 \times 5 = 320$; \$177.147; 743008370688 grains; $(\frac{2}{9})^8 = \frac{256}{43046721}$, $\times 9 = \frac{256}{4782969}$; $3^{12} \times 1 = 531441$ cts., cost of the whole. $531441 \div 20 =$ \$265.72, cost of each.

¶ 200. Formula, $s = \frac{lr - a}{r - 1}$.

136. (p. 211.) $3^8 \times 2 = 13122$ last term. $\frac{13122 \times 3 - 2}{3 - 1} = 19682$ Ans.

137. $\frac{1}{243} \times \frac{1}{3} = \frac{1}{729}$. $9 - \frac{1}{729} = \frac{6561}{729} - \frac{1}{729} = \frac{6560}{729}$. $1 - \frac{1}{3} = \frac{2}{3}$. $\frac{6560}{729} \div \frac{2}{3} = \frac{3280}{243} = 13\frac{121}{243}$, Ans.

138. Ratio $= \frac{1}{2}$, last term $\times$ ratio $= 0$. $1 - \frac{1}{2} = \frac{1}{2}$. $4 \div \frac{1}{2} = 8$, Ans.

139. Ratio $= \frac{1}{4}$. $1 - \frac{1}{4} = \frac{3}{4}$. $5 \div \frac{3}{4} = 6\frac{2}{3}$, Ans.

140. $(\frac{2}{3})^4 \times 8 = \frac{16}{81} \times 8 = \frac{128}{81}$ last term. $\frac{128}{81} \times \frac{2}{3} = \frac{256}{243}$. $8 - \frac{256}{243} = \frac{1688}{243}$. $1 - \frac{2}{3} = \frac{1}{3}$. $\frac{1688}{243} \div \frac{1}{3} = \frac{1688}{81} = 20\frac{68}{81}$, Ans.

COMPOUND INTEREST BY SERIES.

¶ 202. 141–144. (p. 212.) $1.05^5 \times 10 =$ \$12.76; \$4.26; \$27.66; \$510.51.

¶ 204. 145–159. (p. 213.) For 3 yr., \$718.93, \$750.34, \$771.78; for 7 yr., \$857.34, \$947.29, \$1011.64; for 13 yr., \$1116.48, \$1343.74, \$1518.20; for 5 yr. 8 mo., amount for 5 yr. \$785,094. Int. on \$785.094 for 8 mo. at $4\frac{1}{2}$ per cent., \$23.552. \$785.094 + \$23.552 = \$808.65 Ans. \$876.81, \$924.84; for 10 yr. 6 mo. 15 dy., amount for 10 yr. \$978.37. Int. on \$978.37 for 6 mo. 15 dy. at $4\frac{1}{2}$ per cent., \$23.85. \$978.37 + \$23.85 = \$1002.22 Ans; \$1164.90, \$1286.30.

160–174. Amount of \$1 at compound interest for 29 yr. at 5 per cent. = \$4.116, (¶ **201**, and Table I., p. 216.) Then \$1000 ÷ 4.116 (¶ **203**. II.) = \$242.95 Ans. \$184.56,

$140.56 ; for 50 yr., $87.20, $54.29, $33.95 ; for 38 yr. 3 mo. Amount of $1 for 38 yr. (5 per cent.) 6.3854. Int. on $6.3854 for 3 mo. = .0798, and 6.3854 + .0798 = $6.465, amount of $1 for 38 yr. 3 mo. Then 1000 ÷ 6.465 = $154.67 Ans., $107.62, $75.14 ; for 17 yr., $436.30, $371.36, $316.57 ; for 11 yr. 10 mo. 24 dy., (amount of $1. for that time at 5 per cent. 1.7873, at 6 per cent. 2.0008, at 7 per cent. 2.2374.) $559.50, $499.80, $446.94.

175–189. $300 ÷ 150 = 2. (**¶ 203.** III.) In Table I. we find, under 3 per cent., the nearest amount of $1. next less than $2, to be $1.973586, opposite 23 years. The time in this example will, therefore, be 23 yr., and a fraction of a year. $2. — $1.973586 = .026414. We are now to ascertain how long it will take to gain .026414. The amount of $1 for 24 yr., (Table I., p. 216,) is $2.032794. Deducting from this the amount for 23 yr., (2.032794 — 1.973586,) we obtain .059208, which is the sum gained during 1 yr., or 12 mo. Hence, *as the sum gained in 12 months is to the sum for which we wish to ascertain the time, so is 12 months to that time.*

Thus, .059208 : 026414 : : 12 mo. : 5 mo. 11 dy., and 23 yr. 5 mo. 11 dy. = Ans. at 3 per cent.

(176.) Nearest amount, at 5 per cent., $1.979932, (for 14 yr.,) $2 — $1.979932 = .020068. Amount for 15 yr., $2.078928. Then $2.078928 — $1.979932 = .098996. Hence, .098996 : .020068 : : 12 mo. : 2 mo. 13 dy., and 14 yr. 2 mo. 13 dy. = Ans. at 5 per cent. **(177.)** At 6 per cent., 11 yr. 10 mo. 21 dy. ; 37 yr. 1 mo. 29 dy., 22 yr. 6 mo. 4 dy., 18 yr. 10 mo. 6 dy. ; 14 yr. 5 mo. 15 dy., 8 yr. 9 mo. 2 dy., 7 yr. 3 mo. 29 dy. ; 48 yr. 3 dy., 29 yr. 1 mo., 24 yr. 4 mo. 5 dy. ; 6 yr. 2 mo., 3 yr. 8 mo. 24 dy., 3 yr. 1 mo. 15 dy.

190–194. $2297.40 ÷ $400 = $5.7435, (**¶ 203.** IV.) In Table I., opposite 30 yr., we find $5.743491 (differing from the above quotient only by an unimportant fraction, which is disregarded) under 6 *per cent.* Ans. ; **(191.)**

Opposite 35 yr., in Table I., the *nearest* sum to the above quotient is $5.516015, under 5 per cent. Interest on $5.516015 for 9 mo. 27 dy., = .227535, and $5.516015 + .227535 = $5.7435. Hence the Ans. is 5 per cent.; 4; 7; 4½.

195–200. 3; 7; 4½; 6; 4; 5.

ANNUITIES AT COMPOUND INTEREST.

¶ 206. 201–210. (p. 214.) $1.06^{29} \times 100 = 541.8388$, the last term at the series, or rather the amount of the first payment at compound interest. Then by **¶ 199**, 541.8388 × 1.06 = 574.349128, — 100 = 474.349128. The ratio less 1 = 1.06 — 1.00 = .06. Therefore 474.349128 ÷ .06 = $7905.82, Ans. for 30 yr.; $1318.08; $7363.98; $2327.60; $15476.20; $29033.59; $13590.42; $1888.21; $6370.58; $21274.35.

211–220. $3663.90; $1418.625; $6393.05; $14879.68; $9710.35; $12659.57; $7970.84; $2486.53; $4961.95; $51342.76.

221–230. $21561; $7233.53; $30509.78; $63381.04; $11265.93; $55329.32; $77196.67; $23621.85; $16000.95; $28089.84.

231–240. (p. 215.) $4196.92; $7485.82; $145167.95; $39529.09; $36819.90; $31852.88; $55717.39; $42400.84; $23497.91; $93753.79.

¶ 208. 241–250. (p. 215.) The amount of an annuity of $1. for 5 yr., at 5 per cent., (**¶ 205** and Table II. p. 217,) = 5.525631. Then $700 × 5.525631 = $3867.9417, amount of $700 for 5 yr., &c. Amount of $1, at *compound interest*, for 5 yr., at 5 per cent., (Table I.,) = $1.276282, and $3867.9417 ÷ $1.276282 = $3030.64 Ans. for 5 yr.; $4524.25; $5814.49; $8723.55; $4050.46; $5405.21; $1906.28; $7265.76; $3552.98; $4975.47.

251–260. $28.41; $40.46; $21.35; $9.23; $31.68; $52.31; $34.76; $24.98; $17.53; $82.74.

261–270. $20453.38; $11776.14; $4276.82; $17324.16; $18351.87; $9935.66; $15539.60; $22023.73; $12618.99; $18822.53.

271–276. $3797.79; $4126.05; $3506.88; $3956.90; $3648; $3248.28.

277–280. $7026.31; $5018.80; $3011.28; $6524.43.

¶ 210. 281–290. (p. 219.) $3333.33; $7111.11; $\$1111\frac{1}{9}$; $1388.89; $\$2083\frac{1}{3}$; $\$4444\frac{4}{9}$; $\$22222\frac{2}{9}$; $\$666\frac{2}{3}$; $\$3888\frac{8}{9}$; $2777.77.

291–300. $5625; $11250; $\$6428\frac{4}{7}$; $\$8181\frac{9}{11}$; $5000; $3600; $7500; $9473.68; $10000; $4500.

301–310. $12500; $\$10714\frac{2}{7}$; $9375; $\$8333\frac{1}{3}$; $15000; $13636.36; $10000; $\$16666\frac{2}{3}$; $18750; $11538.46.

311–320. £12500; £3750; £5625; £3125; £1250; £250; £1575; £475; £775; £1175.

¶ 211. 321–330. (p. 219.) Present worth of $1 for 8 yr. at 6 per cent. (¶ **207**, and Table III., p. 218,) = $6.20979. This sum × 1000 = $6209.79, the value of the annuity at its commencement 3 years hence. Amount of $1 (*at comp. int.*) for 3 yr. (Table I.) = $1.191016, and $6209.79 ÷ $1.191016 (¶ **203**. II.) = $5213.86 Ans. for $1000; $2606.93; $1564.16; $782.08; $1303.46; $881.14; $443.18; $2241.96; $2487.01; $62.57.

331–340. (p. 220.) $2145.97; $1992.96; $1147.45; $870.30; $656.72; $285.83; $1380.16; $147.08; $1633.43; $2118.83.

341–350. $300 ÷ .05 (¶ **209**) = $6000, value of the annuity at its commencement. Amount of $1. for 2 yr., (Table I.) = $1.1025, and $6000 ÷ $1.1025 = $5442.18 Ans. for 2 yr.; $4701.16; $4061.04: $4264.09; $2886.10; $3508.08; $3867.65; $5183.03; $4477.29; $2617.78.

¶ 213. 351–360. Present worth of $1. for 20 yr., at 5 per cent., (Table III.,) = $12.46221. Then $40000 ÷ $12.46221 = $3209.70 Ans. for 20 yr.; $3853.69; $3547.96; $2641.82; $2331.13; $2191.07; $2602.06; $2393.59; $2224.57; $5627.60.

361–370. Amount of \$1. for 20 yr., at 6 per cent., (Table II.) = \$36.785592, and \$20000 ÷ \$36.785592 = \$543.69 Ans. for 20 yr.; \$364.53; \$271.59; \$425.57; \$1059.20; \$252.98; \$129.23; \$157.15; \$235.84; \$137.88.

371–380. \$1134.09; \$785.72; \$601.71; \$905.13; \$2117.09; \$564.43; \$310.43; \$368.99; \$529.95; \$328.67.

¶ 215. 381–390. (p. 221.) 120; 40320; 3628800; 720; 362880; 39916800; 5040; 87178291200; 479001600; 6227020800.

391–395. 40320; 479001600; 720; 3628800; 6.

396. 403291461126605635584000000.

397–400. 6; 5040; 39916800; 40320.

XV. MENSURATION.*

¶ 216—224. 1. (p. 224.) $27\times 40=1080$ rods, or 6 A. 3 R.

2–7. 7 A. 2 R. 25 r.; 27 r.; 55 r.; 207 r.; 23 r.; 25 ft.

8. $\frac{16\times 9}{2}=72$. $\frac{11\times 8}{2}=44$. $\frac{19\times 12}{2}=114$. $\frac{13\times 14}{2}=91$. Then $72+44+114+91=321$ r., or 2 A. 1 r.

9. (p. 226.) $40\times 3.1416=125.664$ r., circ., and $40^2\times .7854=1256.64$ sq. r., area. Then $\sqrt{1256.64}=35.45$ r. the length of one side.

10. $150\div 3.1416=47.747$ r., diameter, and $150^2\times .0796=1791$ r., area, and $\sqrt{1791}=42.32$ r., the side.

11. $\sqrt{500\div .7854}=25.23$ r., diameter, and then $\sqrt{500\div .0796}=79.26$ r., circumference.

12. $\sqrt{2150\div .7854}=52.32$ r., diameter, and then $\sqrt{2150\div .0796}=164.37$ r., circumference.

13. $40^2=1600$, and $20^2=400$. Then $1600+400=2000$, and $\sqrt{2000}=44.72$ ft., the hypothenuse. The area $=\frac{40\times 20}{2}=400$ sq. ft.

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* See Appendix, "Mensuration."

14. $90^2 = 8100$, and $27^2 = 729$. Then $8100 - 729 = 7371$, and $\sqrt{7371} = 85.85$ ft., base. Area $= \frac{85.85 \times 27}{2} = 1159$ sq. ft.

15–17. 95.39 ft., 1431 sq. ft.; 55.9 ft.; 160.07 m.

18. $210^2 = 44100$, and $189^2 = 35721$. Then $44100 - 35721 = 8379$, and $\sqrt{8379} = 91.53$ ft., the height of the tree.

19. $25^2 \times .3183 = 198.94$ sq. in.

20. $12^2 \times 3.1416 = 452.39$ sq. ft.

21. 201062400 sq. m.

22. $\sqrt{7000 \div 3.1416} = 47.203$ ft., D., and $\sqrt{7000 \div .3183} = 148.296$ ft., circ.

23. $4 \times 3.1416 \times 40 = 502.656$ ft.

24. $7 \times 3.1416 \times 50 = 1099.56$ ft. area of convex surface. $7^2 \times .7854 \times 2 = 76.9692$ ft., area of the two bases. Then $1099.56 + 76.9692 = 1176.5292$ ft., area of the whole surface.

25. (p. 228.) $\frac{25^3 \times .5236}{231} = 31.334$ gall.$_3$

26. $9 \times 231 = 2079$ cu. in., and $\sqrt{2079 \div .5236} = 15.83$ in.

¶ 225–230. **27.** $10^2 \times .7854 = 78.54$ in., area of base, and $\frac{78.54 \times 18}{231} = 6.12$ galls.

28. $231 \times 5, \div 20 = 57.75$, and $\sqrt{57.75 \div .7854} = 8.57$ in.

29. $250 \div 14 = 17\frac{6}{7}$ sq. in.

30. (p. 229.) $500 \div (9^2 \times .7854) = 7.86$ in.

31. $36^2 \times 30 \times \frac{1}{3} = 12960$ cu. in.

32. $22^2 \times .7854 \times 40 \times \frac{1}{3} = 5068.448$ cu. in.

33. $24^2 \times .7854 = 452.3904$, area of larger base. $18^2 \times .7854 = 254.4696$, area of smaller base. $254.4696 \times 452.3904 = 115119.6041$, and $\sqrt{115119.6041} = 339.29$. Then $254.4696 + 452.3904 = 706.86$, area of the two bases. $339.29 + 706.86 = 1046.15$. The length, 135 ft. = 420 in., and $\frac{1}{3}$ of $420 = 140$. Hence $\frac{1046.15 \times 140}{1728} = 84$ ft. 1309 in.

34. $18^2 \times .7854 = 254.4696$, area of larger base. $12^2 \times 7854 = 113.0976$, area of smaller base. Then $254.4696 \times$

113.0976 = 28779.90103296. The square root of this number = 169.6464. 254.4696 + 113.0976 = 367.5672, sum of the area of the two bases. 169.6464 + 367.5672 = 537.2136. $\frac{1}{3}$ of 3 ft. = 12 in. Hence $\frac{537.2136 \times 12}{231} = 27.9$ galls. Ans.

35. $231 \times 2\frac{1}{2} = 577.5$ cu. in.

36. $12^2 \times .7854 \times 4 = 452.39$ in.

37. (p. 230.) 40 — 30 = 10. $\frac{5}{8}$ of 10 = 6.25. 30 + 6.25 = 36.25, and $36.25^2 \times 50 = 65703.1250, \div 294 =$ 223.48 gall.*

38. 33 — 31 = 2. $\frac{5}{8}$ of 2 = 1.25. 31 + 1.25 = 32.25, and $32.25^2 \times 47 = 48882.9375, \div 359 = 136.16$ gall.

39. $\frac{70 \times 25 \times 12\frac{1}{4}}{95} = 230.26$ tons.

40. $163 \times 31 \times 15\frac{1}{2} = 78321.5, \div 95 = 824.43$ tons.

41. $\frac{113.5 \times 24.75 \times 12.375}{95} = 365.92$ tons.

42. 586.7 tons.

MISCELLANEOUS EXAMPLES.

¶ **232.** **1–12.** (p. 239.) $4 \div 9 = \frac{4}{9}$; $\frac{10}{49}$; $\frac{11}{75}$; $\frac{2}{7}$; $\frac{11}{92}$; $\frac{360}{1673}$; 12.8; $15\frac{53}{88}$; $1\frac{1257}{1300}$; $14\frac{5}{6}$; $24\frac{63}{110}$; $18\frac{95}{153}$.

13. $9\frac{1}{2} + 6 = 15\frac{1}{2}, \div 2 = 7\frac{3}{4}$, the greater. $9\frac{1}{2} - 7\frac{3}{4} = 1\frac{3}{4}$, the less.

14. $14.\dot{7} = 14\frac{7}{9}$, and $11\frac{2}{3} + 14\frac{7}{9} = 26\frac{4}{9}, \div 2 = 13\frac{2}{9}$, the greater. $14\frac{7}{9} - 13\frac{2}{9} = 1\frac{5}{9}$, the less.

15. $27.\dot{6}\dot{2} = 27\frac{62}{99}$, and $9.0\dot{7}\dot{5} = 9\frac{\frac{75}{99}}{10} = 9\frac{75}{990}$. Then $27\frac{62}{99} + 9\frac{75}{990} = 36\frac{695}{990}, \div 2 = 18\frac{695}{1980} = 18\frac{139}{396}$, the greater. $27\frac{62}{99} - 18\frac{139}{396} = 9\frac{109}{396}$, the less.

16–24. $\frac{16}{18 \times 9} = \frac{8}{81}$; $1\frac{41}{71}$; $5\frac{1}{3}$; $53\frac{11}{12}$; $58\frac{1}{3}$; $45\frac{65}{72}$; $\frac{27}{28}$; $16\frac{1}{70}$; $30\frac{5660}{6993}$.

25–27. Reverse the steps of the process. $\frac{4 \times 4.375}{3.5} = 5$; $7\frac{11}{12}$; $21\frac{1}{3}$.

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* See Appendix, "Mensuration."

28–30. (p. 240.) $11\frac{1}{12}$; \$4.375; £5 11 s. 5 d.

31–40. 2112; 744; 4696; 4800; 368; 2280; 216; 940; 168; 43.

41–55. 19.45, 8.05, 78.375; $16\frac{3}{8}$, $1\frac{5}{8}$, $66\frac{3}{8}$; $5\frac{1}{6}$, $3\frac{1}{2}$, $3\frac{11}{18}$; $1\frac{5}{8}$, $\frac{1}{8}$, $\frac{21}{32}$; $2\frac{1}{12}$, $\frac{7}{12}$, 1.

56–60. $\frac{40\times 15\times 9}{18\times 6}=50$ rods; 80 r.; 350 r.; $17\frac{7}{9}$ r.; $17\frac{7}{9}$ r.

61–70. 80 lb.; 180 lb.; 60 lb.; 90 lb.; 120 lb.; $115\frac{1}{5}$ lb.; $57\frac{3}{5}$ lb.; 72 lb.; 96 lb.

71–80. \$51.01; \$43.725; \$61.94; \$29.15; \$21.86; \$36.44; \$54.66; \$65.59; \$45.55; \$38.26.

81–90. 35 A.; 35 A.; 330 A.; 20 A.; 50 A.; 165 A.; 30 A.; 10 A.; 30 A.; $26\frac{2}{3}$ A.

91–100. 1.00 — .07 = .93, and \$300 ÷ .93 = \$322.58; \$294.12; \$291.26; \$279.07; \$283.02; \$297.03; \$315.79; \$306.12; \$307.69; \$320.

101–110. 23 bundles,—for \$92; \$115; \$103.50; \$69; \$80.50; \$109.25; \$368; \$92; \$103.50; \$82.375.

111–120. A. \$55, B. \$62.50, C. \$32.50; A. \$27.50, B. \$31.25, C. \$16.25; \$82.50, \$93.75, \$48.75; \$110, \$125, \$65; \$412.50, \$468.75, \$243.75; \$13.75, \$15.625, \$8.125; \$231, \$262.50, \$136.50; \$93.50, \$106.25, \$55.25; \$720.50, \$818.75, \$425.75; \$137.50, \$156.25, \$81.25.

121–130. (p. 241.) 126 bu.; 100.8 bu.; 168 bu.; $90\frac{18}{25}$ bu.; $181\frac{11}{25}$ bu.; $75\frac{3}{5}$ bu.; $151\frac{1}{5}$ bu.; $161\frac{7}{25}$ bu.; $141\frac{3}{25}$ bu.; $201\frac{3}{5}$ bu.

131–140. One man can do $\frac{1}{10}$, and the other $\frac{1}{8}$, of the work in a day, and both can do $\frac{1}{10}+\frac{1}{8}, =\frac{9}{40}$ of it in a day. Then 1 (the *whole* work) $\div\frac{9}{40}=4\frac{4}{9}$ days; $5\frac{5}{11}$ dy.; 5 dy.; $3\frac{1}{8}$ dy.; $4\frac{2}{7}$ dy.; $2\frac{6}{7}$ dy.; $3\frac{3}{4}$ dy.; $6\frac{2}{3}$ dy.; 6 dy.; $5\frac{5}{6}$ dy.

141–150. \$936; \$748.80; \$585; \$842.40; \$702; \$780; \$1170; \$1029.60; \$655.20; \$1006.20.

151–160. $\frac{18\times 13.5}{9}=27$ sq. yds., and \$1.50 × 27 = \$40.50; \$23.62½; \$20.25; \$16.88; \$27; \$30.375; \$36; \$33.75; \$18; \$25.65.

161–170. 1 day = 86400 sec., and 1553400 ÷ 86400 =

$17\frac{47}{48}$ m.; $89\frac{43}{48}$ m.; $880\frac{47}{48}$ m.; $1078\frac{3}{4}$ m.; $20496\frac{1}{4}$ m.; 64725 m.; 711975 m.; 1558400 m.; 46602000 m.; 14714150 m.

171–180. 900; 108; 100; 300; 675; 50; 135; 45; 75; 9.

181–190. 18; 24; 30; 27; 21; 12; 9; $14\frac{2}{5}$; 20; 28.

191–200. 640; 368; 468; 536; 627; 240; 864; 312; 528; 1132.

201–210. 10 miles = 52800 ft., $\div 7\frac{1}{3} =$ 7200; 2880; 1296; 6120; 1440; 4680; 1260; $97\frac{11}{37}$; 180; 2736.

211–220. (p. 242.) 2 cwt. 3 qr. 14 lb. × 18 = 51.75 cwt., × \$6 = \$310.50; \$258.75; \$414; $\$336.37\frac{1}{2}$; \$375.19; \$284.625; \$252.28; \$465.75; \$207; \$345.

221–230. \$1250; \$2250; \$900; \$703.125; \$3000; \$937.50; \$703.125; \$750; \$1406.25; \$45000.

231–240. $\frac{7.5 \times 99}{10} = 74\frac{1}{4}$ ft.; $31\frac{1}{2}$ ft.; 63.225 ft.; $65\frac{1}{4}$ ft.; $23\frac{1}{16}$ ft.; $63\frac{3}{16}$ ft.; $67\frac{7}{8}$ ft.; $61\frac{5}{16}$ ft.; 75 ft.; 105 ft.

241–250. \$365; \$182.50; \$292; \$219; \$438; \$456.25; \$410.63; \$387.81; \$228.125; \$273.75.

251–260. \$337.50; \$312; \$157.50; \$480; \$350; \$75; \$612.50; \$253.125; \$720; \$253.125.

261–270. 33 yd.; 44 yd.; $16\frac{1}{2}$ yd.; $27\frac{1}{2}$ yd.; 11 yd.; 22 yd.; 25 yd.; 10 yd.; 20 yd.; 8 yd.

271–280. \$185 + \$14 + \$7.50 + \$6. = \$212.50, whole cost. Then, $\frac{\$212.50 + 25}{50} =$ \$4.75; \$4.375; \$4.85; \$4.49; \$4.45; \$4.50; \$4.63; \$4.44; \$5.00; \$5.25.

281–290. $\frac{9\frac{3}{5} \times 5}{\frac{4}{5}}$, or $\frac{48 \times 5 \times 5}{5 \times 4} =$ \$60; \$15; \$120; \$36; \$90; \$164; \$10; \$34.50; \$132; \$75.60.

291–300. \$15; \$5.25; \$14.25; \$2.625; \$7.50; \$3.75; \$1.875; \$6.56; \$5.625; \$4.35.

301–310. 2; 8; 3; 5; 20; 1; 6; 12; 10; 11.

311–320. \$0.50; 25 ct.; $37\frac{1}{2}$ ct.; 44 ct.; 40 ct.; 75 ct.; 64 ct.; 29 ct.; $46\frac{3}{7}$; $62\frac{1}{2}$ ct.

321–330. (p. 243.) 2.875 cwt. = 322 pounds, × $4\frac{1}{2}$ ct.

$=$ \$14.49; \$16.10; \$9.66; \$12.88; \$10.46; \$12.08; \$8.05; \$15.70; \$17.31; \$11.27.

331–340. $24 \times \frac{5}{6} = 20$ oz.; $24 \times \frac{1.00}{1.25} = 19\frac{1}{5}$ oz., &c. 16 oz.; 32 oz., $26\frac{2}{3}$ oz.; $27\frac{3}{7}$ oz., $22\frac{6}{7}$ oz.; $26\frac{2}{3}$ oz., $22\frac{2}{9}$ oz.; $21\frac{1}{3}$ oz., $17\frac{7}{9}$ oz.

341–350. 3 square feet $= 3 \times 144 = 432$ square inches, and $432 \div 13\frac{1}{2} = 32$ inches, or $2\frac{2}{3}$ feet; $10\frac{2}{3}$ ft.; $1\frac{1}{3}$ ft.; $6\frac{2}{3}$ ft.; $3\frac{1}{3}$ ft.; 5 ft.; $13\frac{1}{3}$ ft.; $6\frac{2}{9}$ ft.; $1\frac{7}{9}$ ft.; $1\frac{2}{3}$ ft.

351–360. $21\frac{2}{3}$ lb.; $56\frac{1}{3}$ lb.; $8\frac{2}{3}$ lb.; $32\frac{1}{2}$ lb.; $43\frac{1}{3}$ lb.; $108\frac{1}{3}$ lb.; $16\frac{19}{24}$ lb.; $11\frac{5}{9}$ lb.; $17\frac{1}{3}$ lb.; $86\frac{2}{3}$ lb.

361–370. \$68.75; \$192.50; \$4620; \$1232; \$40; \$10.31$\frac{1}{4}$; \$880; \$5.50; \$66; \$990.

371–380. $3\frac{11}{12}$ miles, $\times 4 = 15\frac{8}{12}$ miles, $= 82720$ feet, $\div 2\frac{1}{2} = 33088$; 24816; 57904; 6204; 7445; 10754; 20680; 4136; 6204; 82720.

381–390. The number $+ \frac{1}{3}$ of itself $= \frac{4}{3}$ of itself, and $180 \div \frac{4}{3} = 135$; 90; $73\frac{1}{2}$; 24; $3\frac{3}{4}$; $1\frac{7}{8}$; $63\frac{3}{4}$; 30; $20\frac{1}{4}$; $24\frac{3}{4}$.

391–400. The number $+ \frac{1}{3} + \frac{1}{4}$ of itself $= \frac{19}{12}$ of itself. Then $28 - 9 = 19, \div \frac{19}{12} = 12$; 120; 36; 96; 24; 240; 6; 8; 3; 480.

401–410. 100; 150; 450; 200; 360; 90; 120; 9; 8; 40.

411–420. \$0.01; \$0.02; 6 ct.; 13 ct.; 22 ct.; $6\frac{1}{2}$ ct.; $19\frac{1}{2}$ ct.; .285; $4\frac{1}{3}$ ct.; .0575.

421–430. \$0.001; \$0.002; .003; .004; .0005; $\frac{5}{6}$ mill.; $1\frac{1}{6}$ mill.; $3\frac{1}{6}$ mill.; $4\frac{5}{6}$ mill.; .001833 +.

431–440. In one hour the minute hand passes over 12 spaces of the dial, or its entire circumference, and in the same time the hour hand passes over 1 space; consequently the minute hand gains 11 spaces in one hour. As it must gain 12 spaces, or the whole circuit, in order to overtake it, the following statement will give the required time: —

11 spaces : 12 spaces : : 1 hour : 1 h. 5 min. $27\frac{3}{11}$ sec., or 5 minutes, $27\frac{3}{11}$ seconds past one o'clock, Ans. Again, 1 hour, $= 60$ m., $\div$ 11 (spaces) $= 5\frac{5}{11}$ m., the time necessary for the minute hand to gain 1 space upon the hour hand.

Hence, $5\frac{5}{11}$ m. multiplied by the number representing any hour, will give the time when the hands will next be together after that hour. Thus, between 3 and 4, $5\frac{5}{11} \times 3 = 16\frac{4}{11}$ m., or 16 m. $21\frac{9}{11}$ sec. past 3; 38 m. $10\frac{10}{11}$ sec. past 7, or 21 m. $49\frac{1}{11}$ sec. before 8; 10 m. $54\frac{6}{11}$ sec. past 2; 27 m. $16\frac{4}{11}$ sec. past 5; 32 m. $43\frac{7}{11}$ sec. past 6; 49 m. $5\frac{5}{11}$ sec. past 9; 43 m. $38\frac{2}{11}$ sec. past 8; 21 m. $49\frac{1}{11}$ sec. past 4; 54 m. $32\frac{8}{11}$ sec. past 10.

441–450. (See ¶ **212.**) Present worth of $1 for 10 yr.; (Table III. p. 218.) $7.36009. Then $40000000 ÷ $7.36009 = 5434716; $3584834; $2871689; $2537772; $21817507.46; $3817792.06; $2658460.88; $29059567.03; $2588020.18; $4118510.13.

451. $300 ÷ .06 (¶ **209**) = $5000, the value of the property. Present worth of $1 for 20 yr., at 6 per cent. (Table III.) = $11.46992, and $5000 ÷ $11.46992 = $435.92 Ans.

452–454. $13\frac{1}{3}$ yd.; $7\frac{11}{12}$ yd.; $19\frac{7}{12}$ yd.

455–457. $190.625; $167.75; $160.125.

458–460. $\frac{\$500 \times 4\frac{2}{3}}{7\frac{1}{2}} = \$311.11$; $198.57; $823.53.

461–463. $393.75; $46.82; $7.41.

464–466. $447.75; $397.75; $322.75.

467–469. £1 1 s. $3\frac{3}{4}$ d.; £8 18 s. 3 d.; £26 8 s. $11\frac{1}{4}$ d.

470–472. $\frac{41}{764}$; $\frac{591}{382}$; $\frac{67}{764}$.

473–475. $87\frac{28}{31}$ lb.; $127\frac{13}{31}$ lb.; $268\frac{76}{93}$ lb.

476–478. $\frac{47}{10}$, $\frac{4}{10}$; $\frac{249}{300}$, $\frac{100}{300}$; $\frac{1750}{2000}$, $\frac{1375}{2000}$.

479–481. $700; $2450; $919.52.

482–484. 13 ct.; $12\frac{1}{2}$ ct.; $11\frac{3}{4}$ ct.

485–487. (p. 245.) $31.41; $45.96; $3.68.

488–490. 5 s. 6 d. = £.275. Then $\frac{£137.275 \times 40 \times 1.10}{9} = \$671.12$; $655.87; $669.60.

491–493. $529.81; $320.91; $443.43.

494–496. $100 \div 2\frac{7}{9} = 36$. $\sqrt{36} = 6$; 9; 42.

497–499. $\frac{25 \times 120 \times 6 \times 10}{75 \times 12 \times 8} = 25$; 8; 8.

500–502. $1242.50 — $875 = $367.50. Int. of $875 for 5 yr. 3 mo., at 1 per cent., = 45.9375, and $367.50 ÷ 45.9375 = 8 per cent.; 9; $16\frac{1}{14}$.

503–505. At 20 per cent. loss, the shoes sell at 80 per cent. of the cost; whereas they should sell at 120 per cent. Then $\frac{120}{80} \times 75 = \$1.125$; $\$1.40\frac{5}{8}$; $.62\frac{1}{2}$.

506–508. 120; 5040; 39916800.

509–511. $11.62; $11.14; $18.07.

512–514. $\frac{16}{2085}$; $\frac{112}{6255}$; $\frac{532}{6255}$.

515–517. $3000, $2000; $2177.42, $2822.58; $3395.06, $1604.94.

518–520. £$9\frac{1}{16}$; £49 9 s. $9\frac{1}{4}$ d.; £17 10 s. 6 d.

521–523. £10 14 s. $7\frac{1}{2}$ d.; £4 14 s. $8\frac{1}{4}$ d.; £8 7 s. $3\frac{3}{8}$ d.

524–526. Assume 1 lb. at $.50 and 2 lb. at $.75. Excess 20 ct. Add 2 lb. at .50, = 3 lb. at .50 and 2 lb. at .75; 1 at 50 ct., and 4 at 75 ct.; 1 lb. of each.

527–529. $\$1.00 - .12\frac{1}{2} = .87\frac{1}{2}$, and $\$641.50 \div .87\frac{1}{2} = \$733.14$; $769.80; $855.33.

530–532. (p. 246.) $13.18; $12.08; $15.37.

533–535. $2556.82; $2483.44; 2414.16.

536–538. $2^9 \times 3 \times 5^2$; $5 \times 7 \times 11^2$; $17 \times 19^2 \times 23$.

539–541. 7821; 31442.4; 535680.

542–544. $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$. $1 - \frac{5}{6} = \frac{1}{6}$, and $8\frac{2}{3} \div \frac{1}{6} = 52$; $103\frac{1}{2}$; 177.

545–547. £116 7 s. 5 d.; £116 12 s. 9 d.; £114 15 s. $9\frac{3}{4}$d.

548–550. A. $\$1666\frac{2}{3}$, B. $2000, C. $\$1333\frac{1}{3}$; A. $2500, B. $3000, C. $2000; A. $1193.33, B. $1432, C. $954.67.

551–553. Assume 4 lb. at $62\frac{1}{2}$ ct., and 3 lb. at $1. Excess $3.00. Add 40 lb. at $62\frac{1}{2}$ ct., = 44 lb. at $62\frac{1}{2}$ ct., and 3 at $1.; assume 10 lb. each, &c., = 10 lb. at $62\frac{1}{2}$ ct., and $10\frac{8}{19}$ lb. at $1.; 10 at $62\frac{1}{2}$ ct., and $104\frac{1}{2}$ at $1.

554–556. $108\frac{1}{2}$; $81\frac{3}{8}$; $72\frac{1}{3}$.

557–559. $3642.09 (¶ **207.**); $3319.76; $4953.95.

560–562. $116.875 the whole gain; $1.375 the gain per yard; $39\frac{2}{7}$ per cent.

563–565. 1 h. 20 m. $= \frac{4}{3}$ h. The first pipe fills $\frac{1}{3}$, and the other $\frac{3}{4}$, of the cistern in 1 hour; and both together $\frac{3}{4} + \frac{1}{3}, = \frac{13}{12}$ of it. $2\frac{1}{4}$ h. $= \frac{9}{4}$ h. The discharging pipe will empty $1 \div \frac{9}{4} = \frac{4}{9}$ of it in 1 hour. Hence $\frac{13}{12} - \frac{4}{9} = \frac{23}{36}$, the portion of the cistern filled at the expiration of 1 hour. Then, 1 (the *whole* cistern) $\div \frac{23}{36} = 1\frac{13}{23}$ h.; $\frac{12}{13}$ h.; $3\frac{2}{11}$ h.

566–568. (p. 247.) 250; 625; 100.

569–571. Assume 1 lb. alloy. Deficiency $15.50. Take $15\frac{1}{2}$ oz., or $\frac{31}{32}$ lb. less, &c. $= \frac{1}{2}$ oz.; $\frac{16}{21}$ oz.; $2\frac{10}{19}$ oz.

572–574. A., B., and C. can do, respectively, in 1 day, $\frac{1}{4}$, $\frac{1}{3}$, and $\frac{1}{5}$ of the work, in all $\frac{47}{60}$. Then $1 \div \frac{47}{60} = 1\frac{13}{47}$ dy.; $2\frac{2}{3}$ dy.; $1\frac{7}{8}$ dy.

575–577. Comparing the second and third shares with the first, the second is equal to $\frac{1}{3} \div \frac{2}{5} = \frac{5}{6}$ of the first, and the third $= \frac{1}{3} \div \frac{4}{15} = \frac{15}{12}$ of it. $1 + \frac{5}{6} + \frac{15}{12} = \frac{37}{12}$. Hence $600 $\div \frac{37}{12} =$ $$194\frac{22}{37}$, 1st share. $\frac{5}{6}$ of 1st $=$ $$162\frac{6}{37}$, 2d, and $\frac{15}{12}$ of 1st $=$ $$243\frac{9}{37}$, 3d; $284.11, $236.76, $355.14; $309.40, $257.84, $386.76.

578–580. In the first place, $375 must be expended to build the wall, and then such an additional sum as shall yield, in 15 yr., at 6 per cent., a *compound interest* of $375, *leaving the principal unchanged.* By Table I., p. 216, we find the *compound interest* of $1 for 15 yr., at 6 per cent., to be ($2.396558 — $1, *the principal,*) $1.396558. Then, $375 ÷ $1.396558 (see ¶ **215**, note, &c.,) = $268.52, the sum to be invested to provide for the *rebuilding* of the wall. Adding the original cost of the wall, $375 + $268.52 = $643.52, the whole investment at 6 per cent.; at 7 per cent., $588.18; at 5 per cent., $722.57.

581–583. 2 h. 21 min. 8 sec.; 36 min. 52 sec.; 1 h. 54 min. 44 sec.

584–586. 60; 61; 62.

587. It must be borne in mind, that from *each end* of the shorter sides, the thickness of the wall must be deducted. The thickness $= 1\frac{1}{2}$ feet, and 110 ft. (the length of the shorter side) — $\overline{1\frac{1}{2}\text{ ft.} \times 2} = 107$ ft. Then $153\frac{3}{4}$ ft. × 8 ft. × $1\frac{1}{2}$ ft. × 2 (2 walls) = 3688 cubic feet, the contents of the longer sides; and 107 ft. × 8 ft. × $1\frac{1}{2}$ ft. × 2 = 2568 cubic feet, contents of the shorter sides. 3688 + 2568 = 6256 cu. ft., the contents of the whole wall. The contents of one brick

$= 8 \times 4 \times 2 = 64$ cu. inches, and 1728 in. (1 cu. ft.) $\div 64 = 27$, number of bricks occupying 1 cubic foot. Hence $6256 \times 27 = 168912$, Ans.

588–590. \$4194.304; 23.32 ft.; A. \$150, B. \$100, C. \$50.

591. (p. 248.) $\frac{£225 \times 40 \times 1.09}{9} = \$1090$, price paid for the stock, and $\$1090 + \$50 = \$1140$, the whole cost of the stock. 85 per cent. of $\$1500 = \$1275$, the price for which the stock was sold, and $\$1275 - \$1140 = \$135$, the gain, Ans.

592. Had he labored the whole time, he would have received $\$1.75 \times 40 = \$70$. Then $\$70 - 45 = \$25$, his whole loss. Now, every idle day occasioned him a loss of \$2.50, that is, his wages \$1.75, and his board \$0.75. Hence $\$25 \div 2.50 = 10$, the number of days he was idle. He worked 30 days.

593–594. 180 apples, at 2 for a cent, cost 90 ct., and 180, at 3 for a cent, cost 60 ct., the whole, $180 + 180 = 360$, cost $.90 + .60 = \$1.50$. Then 360, at $\frac{2}{5}$ ct. each, cost \$1.44, and $\$1.50 - \$1.44 = 6$ ct. lost, Ans.; 2250 men.

595–597. $1 - \frac{1}{4} = \frac{3}{4}$. After $\frac{3}{7}$ were spent, $\frac{4}{7}$ remained. Then, $\frac{4}{7}$ of $\frac{3}{4} = \frac{3}{7}$, and $£820 \div \frac{3}{7} = £1913$ 6 s. 8 d.; \$1920; 150 men.

598. The shares of A. and B. are compared with C.'s share. If, therefore, C. has 1 share, A. will have a like share $+ \$240$, and B. will have \$190 less than such a share; in all 3 shares $+ \$50$. Then $\frac{\$2000 - \$50}{3} = \$650$, C.'s share, &c., A. \$890, B. \$460.

599. $\frac{2}{3}$ of $\frac{4}{5}$ of $1\frac{1}{2} = \frac{4}{5}$, and $1 \div \frac{4}{5} = \frac{5}{4}$. Then $(\frac{5}{4})^2 = \frac{25}{16} = 1\frac{9}{16}$, Ans.

600. $\frac{2}{5}$ of $\frac{4}{9}$ of $\frac{3}{16} = \frac{1}{30}$, the part sold. $\frac{3}{16} - \frac{1}{30} = \frac{37}{240}$, the *part* left. Then $\frac{37}{240}$ of $\$6000 = \$925$, its value.

601. (p. 249.) 95000000 miles $= 501600000000$ feet, $\div 1500 = 334400000$ seconds, $=$ 10 yr. 7 mo.* 8 dy. 8 h. 53 m. 20 sec. Within 10 years, succeeding 1849, there will occur two leap years, viz., 1852 and 1856. Consequently, 1 day for each of those years must be deducted from the days above, = 6 dy. Then 10 yr. 7 mo. 6 dy. 8 h. 53 m. 20 sec., added to 1849 yr. 12 h., ("noon") = A. D. 1859, Aug. 7, at 8 o'clock, 53 m. 20 sec., P. M., or evening.

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* 220 dy. = the first 7 mo. of the yr. and 8 dy.

602–603. 495, 360, 385; in 11 months.

604. In 293 days from April 3, 1848, = Jan. 21, 1849.

605–606. $0.75; in 168 days.

607. A. having lost $\frac{1}{4}$ of his stock, has $\frac{3}{4}$ left. B. having double that amount, has $\frac{6}{4}$. Consequently, he has increased it $\frac{2}{4}$, or $\frac{1}{2}$. $\$500 \div \frac{1}{2} = \$1000$ Ans.

608. 250 men can be supported, and 550 men must leave.

609–611. $33\frac{1}{3}$ per cent.; $859.18; $39.50.

612. $2.5^3 = 15.625$. This number raised to the second power = 244.140625, Ans.

613. (p. 250.) (See ¶ **135**. **291–300**, &c.) £1 N. Eng. = (in Pa. currency,) $\frac{10}{3} \div \frac{8}{3} = \frac{5}{4}$. Then $\frac{5}{4}$ of 10 s. 6 d. = 13 s. $1\frac{1}{2}$ d., Ans.

614–615. 27; 32 feet.

616. $\frac{36 \times 127.5 \times 7 \times 13.5 \times 11 \times 910 \times 7.4 \times 10}{51 \times 140 \times 11 \times 33.75 \times 10.5 \times 15.6} = 740$, Ans.

617. By Fellowship, gain is proportioned to stock *multiplied* by time. Reversing this rule, stocks must be proportioned to gain *divided* by time. Dividing each partner's number representing his proportion of the gain by the number expressing his time, we have $\frac{2}{4}$, $\frac{3}{5}$, and $\frac{4}{13}$. $549 divided in the proportion of those fractions, gives, for A. $195, B. $234, C. $120.

618. C.'s gain being $\frac{1}{3}$ of the whole gain, he must have contributed $\frac{1}{3}$ of the stock, that is $\frac{1}{2}$ as much as A. and B. together. Then $\frac{\$3000 + \$2000}{2} = \$2500$, what C. contributed. A's gain was $600.

619–620. 40 days; $150 is 75 per cent. of the cost, which, therefore, was $200. As I expected to gain 15 per cent., my loss was $1.15 - .75 = 40$ per cent., and .40 of $200 = $80, the whole loss.

621–622. 320 times; $\sqrt{12^2 + 6^2} = 13.4$ feet.

623. Let A.'s share be represented by 1. Then $\frac{3}{10}$ = C.'s, and $1 + \frac{3}{10} - 10$ acres = B.'s. The sum of the whole

$=2\frac{6}{10}$ shares — 10 acres. Then $1500-10\div2\frac{6}{10}=580\frac{10}{13}$ acres, A.'s share, &c., B.'s 745 acres, C.'s $174\frac{3}{13}$ acres.

624. $\frac{1}{5}+\frac{3}{10}+\frac{1}{4}+\frac{1}{8}=\frac{35}{40}$. Hence \$1000 was $\frac{5}{40}$ of the estate, and \$8000 = the whole. A.'s = \$1600, B.'s \$2400, C.'s \$3000, and D.'s \$1000.

625. (p. 251.) $1\times\$1.00=\$1.00$, and $\frac{2}{3}\times.75=.50$, and $\frac{3}{4}$ of $\frac{2}{3}\times.50=.25$. Hence $\$1.00+.50+.25=\$1.75$, the daily wages of three, (one of each kind.) Then $\$42\div\$1.75=24$ at \$1.00, &c., 16 at .75, and 12 at .50.

626. 1 acre = 160 sq. rods, ÷ .7854 (¶ **219**) = 203.7178, nearly. And $\sqrt{203.7178}=14.28$ rods, the diameter of the field. $\frac{1}{2}$ of 14.28 r. = 7.14 rods (nearly), the length of the rope.

627. They used $\frac{5}{3}$, or $1\frac{2}{3}$, loaves each. Hence A. furnished to C. $\frac{1}{3}$ of a loaf, and B. furnished to him $1\frac{1}{3}$, or $\frac{4}{3}$, loaves. The money, 10 cents, must be divided in the ratio of those fractions, that is of 1 to 4, = 2 ct. for A., and 8 ct. for B.

628. $2500\times12=30000$. Sum of the products of the payments by their times = 5800. Then $\frac{30000-5800}{1400}=17\frac{2}{7}$ mo., Ans.

629. At \$5.25 per yard, the cloth is sold at 105 per cent. of its prime cost. $\$525\div\$1.05=\$5.00$. But as the measure is $\frac{1}{4}$ short, \$5.00 must be $\frac{4}{3}$ of the cost. Hence $\$5.00\div\frac{4}{3}=\$3.75$ Ans.

630. B., having gained 50 per cent., has 150 per cent. of his original capital. A., after losing, has $\frac{2}{3}$ as much as B., that is, $\frac{2}{3}$ of 150 per cent. = 100 per cent. of B.'s original capital. A. has, therefore, in losing \$2000, lost $\frac{1}{2}$ his stock, (having had twice as much as B. at the commencement,) which must have been \$4000, and B.'s \$2000.

631. 22 yd. — 17 yd. = 5 yd. By cutting off 5 yd. more from one piece than from the other, the former is made $\frac{1}{4}$ shorter than the latter. Hence 5 yd. is $\frac{1}{4}$ of what was left after cutting off 17 yd. 5 yd. × 4 = 20 yd., + 17 yd. = 37 yd., the original length.

632. 2519, that is, 1 less than the least common multiple.

633. $17\frac{1}{2}$ miles ÷ 5 = $3\frac{1}{2}$ m., A.'s rate per hour, and 8 ÷ 3 = $2\frac{2}{3}$ m., B.'s rate per hour. $3\frac{1}{2}$ m. × 2 = 7 m., and 81 — 7 = 74 m., the distance between A. and B. when B. starts and after A. has travelled 7 miles. $3\frac{1}{2}$ m. + $2\frac{2}{3}$ m. = $6\frac{1}{6}$ m. Then,

$6\frac{1}{6}$: $3\frac{1}{2}$: : 74 : 42, + 7 = 49 miles, A.'s distance.
$6\frac{1}{6}$: $2\frac{2}{3}$: : 74 : 32 miles, B.'s distance.

Or thus, $3\frac{1}{2}$ m. + $2\frac{2}{3}$ m. = $6\frac{1}{6}$ m., and 74 m. ÷ $6\frac{1}{6}$ = 12 hours, the time they will travel after B. sets out. Then $3\frac{1}{2}$ × 12 = 42, + 7 = 49 m., A.'s distance; and $2\frac{2}{3}$ × 12 = 32, B.'s distance, &c., as before.

APPENDIX.

The different sections of the Appendix are referred to, under their appropriate heads, in the body of the Key.

COMPOUND INTEREST.

[See Key, page 47.]

Compound Interest may be computed by multiplying the principal by the *amount* of $1 for 1 year, or for the time for which interest is to be reckoned, making the *product* the next principal, to be multiplied as before, &c. Thus:

1. Required the compound interest of $500 for 4 yr. at 5 per cent. See Arithmetic, p. 145, Ex. 316.

Amount of $1 for 1 year = 1.05, the multiplier.

$500	Principal.
1.05	
2500	
500	
525.00	Amount for 1 yr.
1.05	
2625	
525	
551.25	Amount for 2 yr.
1.05	
275625	
55125	
578.8125	Amount for 3 yr.
1.05	
289405	
57881	
607.7505	Amount for 4 yr.
500.	Deduct the principal.
$107.75	Comp. interest for 4 yr., Ans.

2. What would be the amount of \$160 at 6 per cent. compound interest, payable quarterly, the interest having remained unpaid from Sept. 1, 1851, to Dec. 13, 1852?

The time from Sept. 1, 1851, to Dec. 13, 1852 = 1 yr. 2 mo. 12 dy. Rate for 3 mo. = .015, and the amount of \$1, or *multiplier*, = 1.015. The multiplier for 2 mo. 12 dy. = 1.012. Ans. \$171.85.

BANK DISCOUNT.

[See page 48.]

It is often necessary to ascertain for what amount a bank note must be given in order to obtain a specified sum of money. For example: —

I wish to obtain, from a Bank, \$1000 for 3 months. For what amount must the note be given, the rate of discount being 6 per cent.?

The bank discount (interest) of \$1 for 3 mo.* (93 days) $= \frac{\$1.00 \times 93}{600} = .0155$, and $\$1.00 - .0155 = \$0.9845$, the *present worth* of \$1.00 for 3 mo. Therefore to obtain \$0.9845, a note must be given for \$1.00. Hence $\$1000 \div .9845 = \$1015.74$, Ans., or the sum for which the note must be given to obtain \$1000 for 3 months.

Proof. $\frac{\$1015.74 \times 93}{6000} = \$15.74$, the bank discount on the note. Deducting this discount from the face of the note, we obtain $\$1015.74 - \$15.74 = \$1000$, as before, the sum to be obtained.

Hence we may derive the following

Rule. — *Divide the sum to be obtained by the present worth of* \$1 *for the given time and rate, at bank discount. The quotient will be the face of the note.*

Examples.

1. For what sum must a note be given in order to obtain from a bank, at 6 per cent. discount, \$8000 for 6 months?
Ans. \$8251.68.

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* It must be recollected that 3 days (days of grace) must be added to the time specified in the note.

2. What will be the face of a bank note, on which \$450 is taken out for 1 mo.? Ans. \$452.49.

3. A drover disposed of a lot of cattle, for ready money, for \$23,750, and after making purchases for cash, amounting to \$1300, he went to the bank and took up a note, on which he had previously received \$8000 for 3 months. After reserving \$230 for personal expenses, he made a purchase of merchandise amounting to \$20,000, for which he paid the remainder of his money and obtained the balance from the bank for 6 months. For what amount was his note drawn? Ans. \$6091.75.

4. Find the amounts for which notes must be given, respectively, to obtain the following sums for the times specified.

\$400 for 3 months. Ans. \$406.30.
\$2000 for 6 months. Ans. \$2062.92.
\$600 for 2 months. Ans. \$606.37.
\$100 for 4 months. Ans. \$102.09.

5. A gentleman has a note in a bank, on which he received \$575 for 3 mo. at 4 per cent. discount; and in order to take up the note, he goes to another bank and obtains the money for 6 months at 6 per cent. For what amount was the last named note given? Ans. \$599.28.

DUODECIMALS.

[See page 57.]

The following examples are inserted for the purpose of giving the pupil some additional exercises in operations in Duodecimals, and especially in the practical application of the rule in its most common uses.

1. Required the contents of a load of wood 9 ft. 6 in. long, 4 ft. wide, and 4 ft. high. Ans. 152 cu. ft., = $1\frac{3}{16}$ cords.

2. What are the contents, in cubic feet, cord feet, and cords, of a pile of wood 18 ft. 6 in. long, 4 ft. wide, and 5 ft. 6 in. high? And what is its value at \$4.50 per cord? Ans. 407 cu. ft.; $25\frac{7}{16}$ cd. ft.; $3\frac{23}{128}$ cords. Whole value, \$14.31.

3. Required the contents of a stock of 12 boards, each 14 ft. 6 in. long, and 13 in. wide at one end and 7 in. at the other.

Ans. 145 sq. ft.

4. Calculate the separate, and the whole, contents of 4 loads of wood of the following dimensions:—

8 ft. long,	5 ft. 3 in. high,	and 4 ft.	wide.	Ans. $1\frac{5}{16}$ cords.
12 ft. "	4 ft. "	and 4 ft. 2 in.	"	Ans. $1\frac{9}{16}$ cords.
9 ft. "	3 ft. 4 in. "	and 3 ft. 6 in.	"	Ans. $6\frac{9}{16}$ cd. ft.
16 ft. "	3 ft. "	and 3 ft. 6 in.	"	Ans. $1\frac{5}{16}$ cords.

Whole contents, 5 cords, 1 cu. ft.

5. What is the number of square yds. of plastering in a room of the following dimensions, and how many rolls of paper, 9 yd. long and 2 feet wide, would be required to hang the walls? 15 ft. 6 in. long, 14 ft. wide, and 10 ft. high. The room contains 4 windows, each 3 ft. 6 in. by 5 ft. 8 in.; 2 doors, each 6 ft. 4 in. by 2 ft. 6 in.; one fireplace, 3 ft. by 3 ft. 6 in., and a mopboard 8 in. wide, extending around the room.

Operation.

15 ft. 6 in. × 14 ft. = 217 ft., sq. contents of ceiling.
15 ft. 6 in. × 10 ft. × 2 = 310 ft., " " of side walls.
14 ft. × 10 ft. × 2 = 280 ft., " " of end walls.
807 ft. whole contents.

3 ft. 6 in. × 5 ft. 8 in. × 4 = 79 4′ contents of windows.
6 ft. 4 in. × 2 ft. 6 in. × 2 = 31 8′ " of doors.
3 ft. × 3 ft. 6 in. = 10 6′ " of fireplace.
50 ft. 6 in.* × 8 in. = 33 8′ " of mopboard.
155 2′ whole contents to be deducted.

Then 807 ft. — 155 2′ = 651 10′, ÷ 9 (sq. ft.) = $72\frac{23}{54}$ sq. yd. of plastering.

Then 651 10′ (the whole clear contents) — 217 ft. (the contents of the ceiling) = 434 10′, = $48\frac{17}{54}$ sq. yd., contents of the walls to be hung with paper.

1 roll of paper = 9 × 3 × 2 = 54 sq. ft., = 6 sq. yd. Then $48\frac{17}{54}$ sq. yd. ÷ 6 = $8\frac{17}{324}$ rolls of paper.

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* Length of the mopboard, after deducting the width of the fireplace and doors.

MENSURATION OF BOARDS AND TIMBER.

Measurers of lumber usually have, for the purpose of abbreviating their computations, short practical rules, which are too numerous to introduce into an elementary treatise on arithmetic, and which vary so much in different localities that they are better learned by practice than otherwise. The following are of quite general use: —

MEASUREMENT OF BOARDS.

RULE. — *Multiply the length in feet by the width in inches, and divide the product by* 12. *The quotient will be the contents in feet.*

Examples.

1. Required the contents of a board 16 ft. long and 8 in. wide. Ans. 10⅔ ft.

2. What will be the contents of a board 22 ft. long and 17 in. wide? 28 ft. long and 22 in. wide? 10 ft. long and 8 in. wide? Ans. 31⅙ ft., 51⅓ ft., 6⅔ ft.

BOARD MEASURE OF TIMBER, JOIST, &c.

RULE. — *Multiply together the width and depth in inches, and that product by the length in feet, and divide the last product by* 12. *The quotient will be the contents in square feet.*

Examples.

1. What is the board measure of a plank 30 ft. long, 18 in. wide, and 3 in. thick? Ans. 135 ft.

2. Required the contents of a piece of joist 16 ft. in length, 4 in. wide, and 4½ in. deep. Ans. 24 ft.

3. What is the number of square feet in a stick of timber 42 ft. long, 10 in. wide, and 16 in. deep? Ans. 560 ft.

A TABLE OF ROOTS AND POWERS.

[See Arithmetic, Involution and Evolution, Page 193, and Key, Page 68, &c.]

Powers										Powers
1st	1	2	3	4	5	6	7	8	9	1st
2d	1	4	9	16	25	36	49	64	81	2d
3d	1	8	27	64	125	216	343	512	729	3d
4th	1	16	81	256	625	1296	2401	4096	6561	4th
5th	1	32	243	1024	3125	7776	16807	32768	59049	5th
6th	1	64	729	4096	15625	46656	117649	262144	531441	6th
7th	1	128	2187	16384	78125	279936	823543	2097152	4782969	7th
8th	1	256	6561	65536	390625	1679616	5764801	16777216	43046721	8th
9th	1	512	19683	262144	1953125	10077696	40353607	134217728	387420489	9th
10th	1	1024	59049	1048576	9765625	60466176	282475249	1073741824	3486784401	10th
11th	1	2048	177147	4194304	48828125	362797056	1977326743	8589934592	31381059609	11th
12th	1	4096	531441	16777216	244140625	2176782336	13841287201	68719476736	282429536481	12th

MENSURATION.

[See pages 77 and 79.]

It is often a matter of complaint among pupils that the rules given for the operation of examples in Mensuration are arbitrary, and carry along with them no explanation of the principles upon which they are based. This must, of course, be the case, to a considerable extent, so long as the rules are prepared for those unacquainted with the higher branches of mathematics, upon which the demonstration of those rules depends. There are some of them, however, that will admit of a partial explanation, and it must be left to the teacher's judgment to decide when this explanation is necessary, and to what extent it can be appreciated. The rule for gauging casks, given on page 230 of the arithmetic, may be taken as an example.

EXPLANATION. — It is found that in bodies shaped like ordinary casks, if to the smaller diameter about *five eights* of the difference of the two diameters be added, the sum will be the mean diameter. The cask then becomes a cylinder. Now, by ¶ **225** and ¶ **219**, the

$(\text{Diameter})^2 \times .7854 \times$ the length (in inches) $=$ the solid contents of the cylinder, in cubic inches, which may be reduced to gallons by dividing by the number of inches in a gallon, viz., 231 for wine or common liquid measure, and 282 for ale or beer measure. Then, —

$$\frac{(\text{Diameter})^2 \times .7854 \times \text{length}}{231} = \text{number of wine gall.}$$

The decimal .7854 is contained in 231 (inches) 294 times, and may be cancelled thus: —

$$\frac{(\text{Diameter})^2 \times \cancel{.7854} \times \text{length},}{\times \cancel{.7854} \times 294} \text{ or } \frac{(\text{Diameter})^2 \times \text{length}}{294} =$$

contents in galls., &c. The formula for beer measure may be found in a similar manner. (See page 79, Ex. 37, &c.)

END OF THE KEY.

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